CS70: Lecture 9. Outline.

1. Public Key Cryptography
2. RSA system
   2.1 Efficiency: Repeated Squaring.
   2.2 Correctness: Fermat's Theorem.
   2.3 Construction.
3. Warnings.

Isomorphisms.

Bijection:

Isomorphisms.

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$f(x) = ax \pmod{m}$ if $gcd(a,m) = 1$.

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Simplified Chinese Remainder Theorem:

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Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{m}n$.

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Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

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Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

Consider $(a', b') = (2,4)$, then $x = 22 \pmod{45}$.

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Now consider:

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Now consider: $(a,b) + (a',b') = (0,2)$.

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What is x where $x = 0 \pmod{5}$ and $x = 2 \pmod{9}$?

Try $43 + 22 = 65$

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What is x where $x = 0 \pmod{5}$ and $x = 2 \pmod{9}$?

Try $43 + 22 = 65 = 20 \pmod{45}$.

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Isomorphism:
the actions under $\pmod{5}$, $\pmod{9}$

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Is it $0 \pmod{5}$? Yes! Is it $2 \pmod{9}$? Yes!

Isomorphism:
the actions under $\pmod{5}$, $\pmod{9}$
correspond to actions in $\pmod{45}$!

Xor

Computer Science:

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  1 - True
  0 - False

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$1 \vee 1 = 1$

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$A \oplus B$ - Exclusive or.

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Note: Also modular addition modulo 2!

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Property: $A \oplus B \oplus B = A$.

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By cases: $1 \oplus 1 \oplus 1 = 1$.

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Cryptography ...

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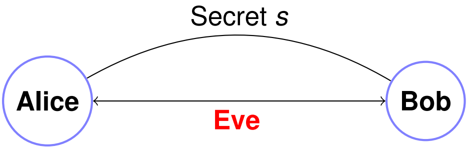
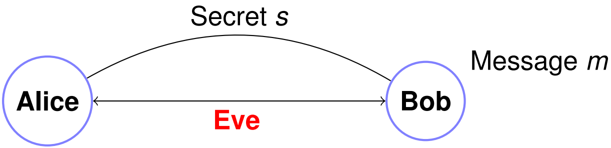


Cryptography ...



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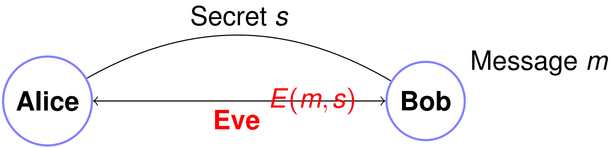

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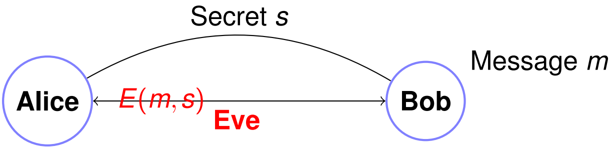

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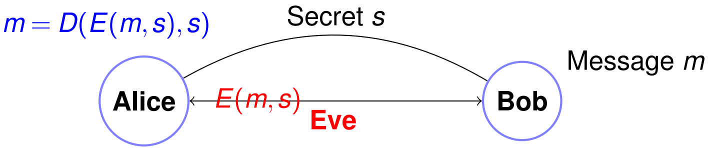

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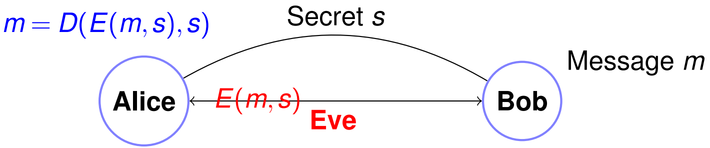
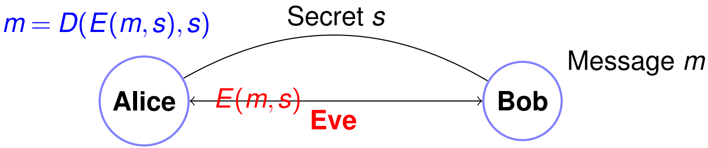


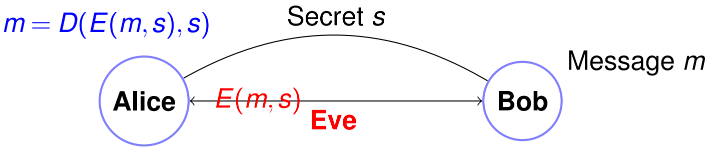
Example:

Cryptography ...



Example:
One-time Pad: secret s is string of length $|m|$.

Cryptography ...



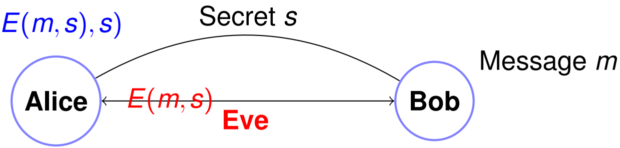
Example:

One-time Pad: secret s is string of length $|m|$.

$m = 1010101110101101$

Cryptography ...

$m = D(E(m,s),s)$



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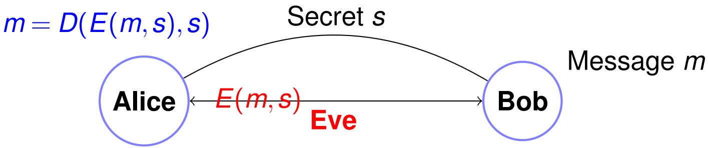

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One-time Pad: secret s is string of length $|m|$.

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$E(m,s)$ – bitwise $m \oplus s$.

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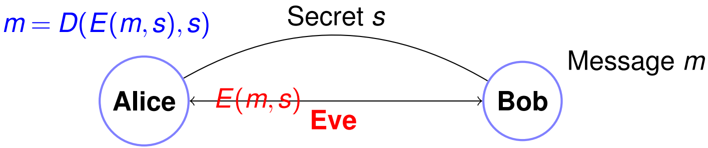

Example:

One-time Pad: secret s is string of length $|m|$.

$m = 10101011110101101$

$s = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$E(m,s)$ – bitwise $m \oplus s$.

$D(x,s)$ – bitwise $x \oplus s$.

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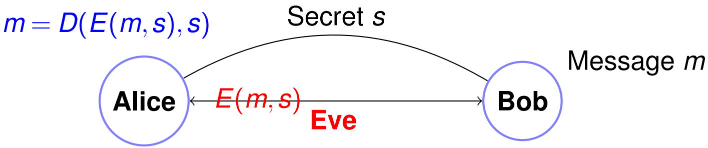

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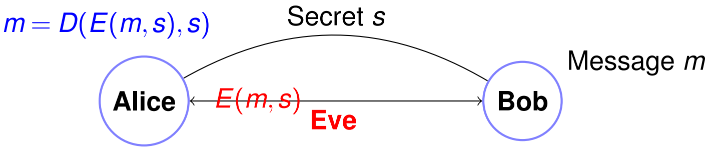
$s = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$
$E(m,s)$ – bitwise $m \oplus s$.
$D(x,s)$ – bitwise $x \oplus s$.
Works because $m \oplus s \oplus s = m$!

Cryptography ...



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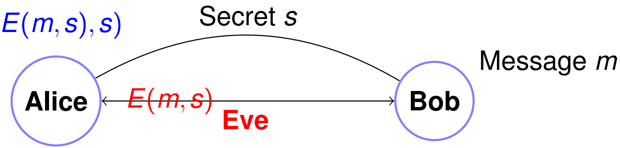

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...given $E(m,s)$ any message m is equally likely.

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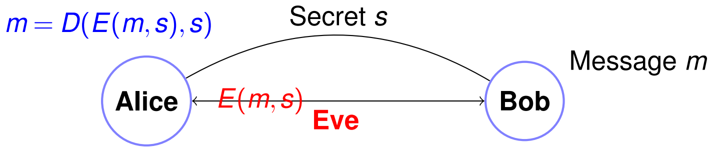

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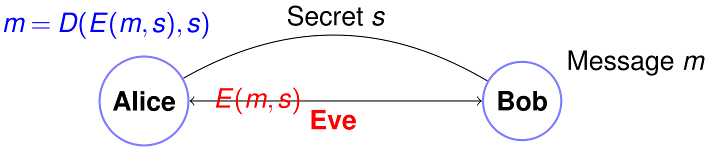
Cryptography ...



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Shared secret!

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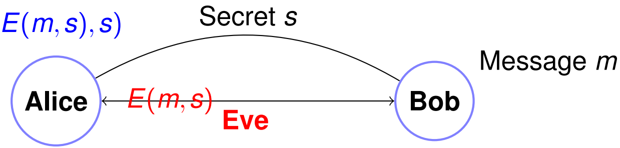

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Shared secret!

Uses up one time pad..

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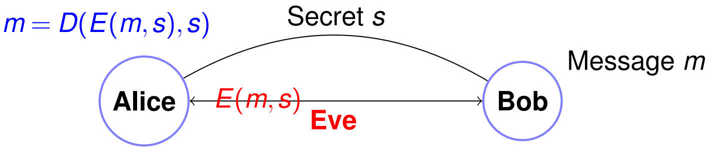

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Works because $m \oplus s \oplus s = m$!
...and totally secure!
...given $E(m,s)$ any message m is equally likely.

Disadvantages:

Shared secret!

Uses up one time pad..or less and less secure.

Public key crypography.

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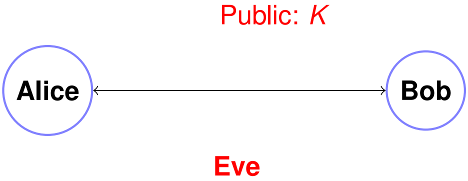

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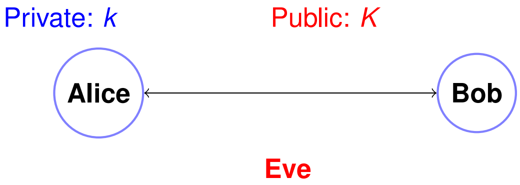

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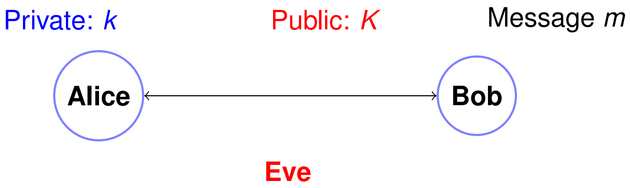

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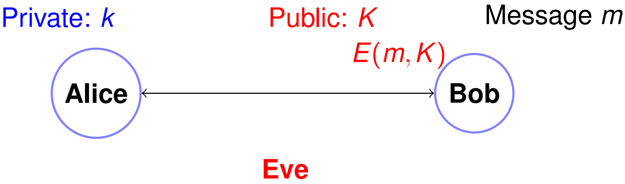

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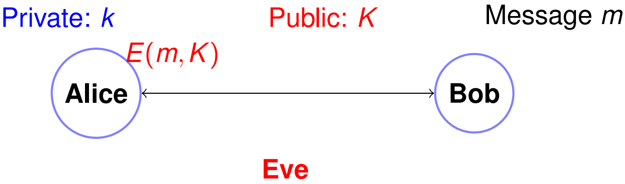
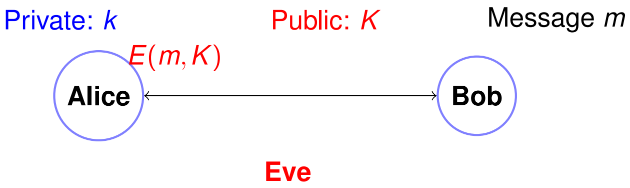


Public key crypography.

$m = D(E(m,K),k)$



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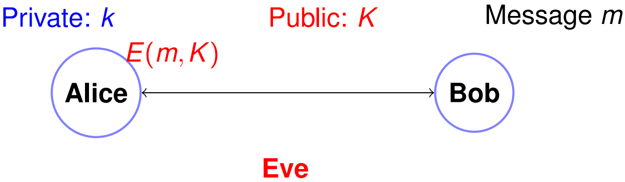

Everyone knows key K!

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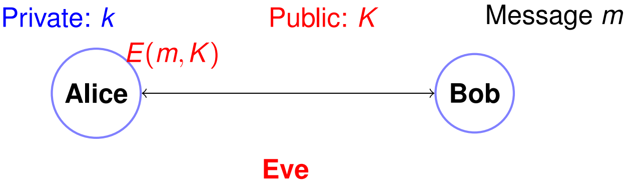

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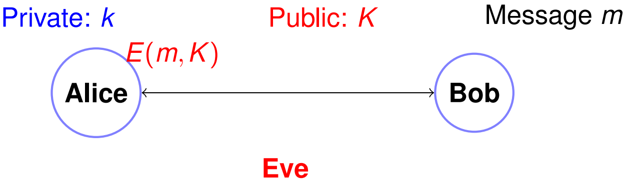

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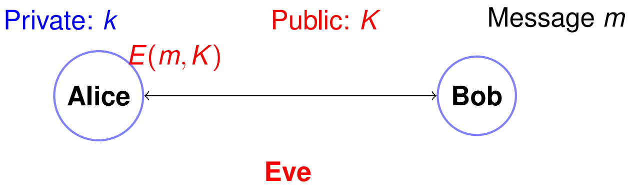

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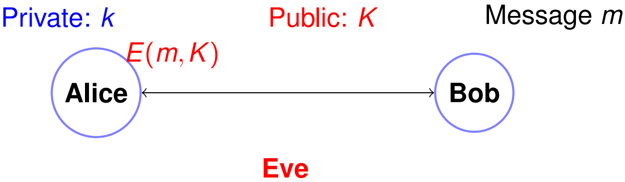

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Bob (and Eve and me and you and you ...) can encode.

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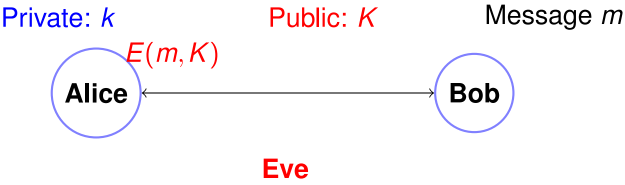

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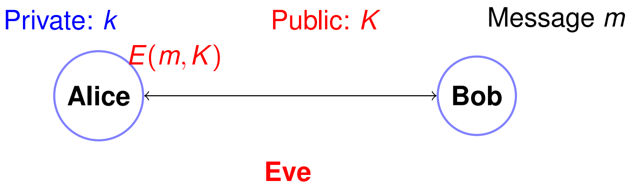

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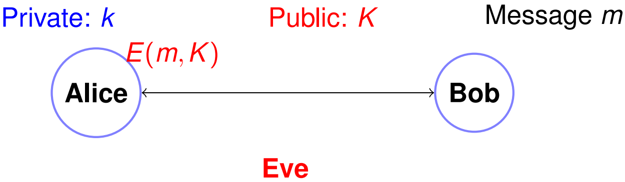

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Is this even possible?

Is public key crypto possible?

[1] Typically small, say $e = 3$.

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We don't really know.

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Iterative Extended GCD.

Example: $p = 7$, $q = 11$.

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$$7(0) + 60(1) \quad = \quad 60$$

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$$\begin{aligned} 7(0) + 60(1) &= 60 \\ 7(1) + 60(0) &= 7 \end{aligned}$$

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Confirm:

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$$

Confirm: $-119 + 120 = 1$

$d = e^{-1} = -17 = 43 = \pmod{60}$

Encryption/Decryption Techniques.

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Public Key: $(77, 7)$

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Message Choices: $\{0,\ldots,76\}$.

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$E(2)$

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$E(2) = 2^e = 2^7 \equiv 128 \pmod{77}$

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$E(2) = 2^e = 2^7 \equiv 128 \pmod{77} = 51 \pmod{77}$

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Obvious way: 43 multiplications.

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In general, $O(N)$ or $O(2^n)$ multiplications!

Repeated squaring.

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Notice: $43 = 32 + 8 + 2 + 1$.

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4 multiplications sort of...

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Need to compute $51^{32} \ldots 51^{1}$.?

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$51^1 \equiv 51 \pmod{77}$
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Need to compute $51^{32} \ldots 51^{1}$.?

$51^1 \equiv 51 \pmod{77}$

$51^2 = (51) * (51) = 2601 \equiv 60 \pmod{77}$

$51^4 = (51^2) * (51^2) = 60 * 60 = 3600 \equiv 58 \pmod{77}$

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$51^{16} = (51^8) * (51^8) = 53 * 53 = 2809 \equiv 37 \pmod{77}$

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5 more multiplications.

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5 more multiplications.

$51^{32} \cdot 51^{8} \cdot 51^{2} \cdot 51^{1} = (60) * (53) * (60) * (51) \equiv 2 \pmod{77}$.

Repeated squaring.

Notice: $43 = 32+8+2+1$. $51^{43} = 51^{32+8+2+1} = 51^{32} \cdot 51^{8} \cdot 51^{2} \cdot 51^{1} \pmod{77}$.

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Repeated Squaring took 9 multiplications versus 43.

Recursive version.

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(define (power x y m)
  (if (= y 1)
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Claim: Program correctly computes x^y.

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Note: $y/2$ is integer division.

Repeated Squaring: x^y

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Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute x^1,

Repeated Squaring: x^y

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Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute $x^1, x^2, x^4, \ldots, x^{2^{\lfloor \log y \rfloor}}$.
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Conclusion: $x^y \mod N$

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Conclusion: $x^y \mod N$ takes $O(n^3)$ time.

RSA is pretty fast.

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Similar, not same, but useful.

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Proof: Consider $S = \{a \cdot 1, \ldots, a \cdot (p-1)\}$.

All different modulo p since a has an inverse modulo p.
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All steps are polynomial in $O(\log N)$, the number of bits.

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Breaking in general sense $\implies$ factoring algorithm.

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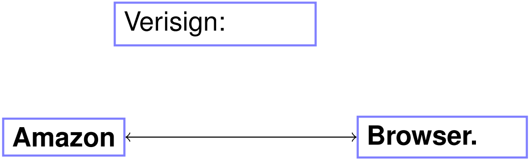
CS161...

Signatures using RSA.



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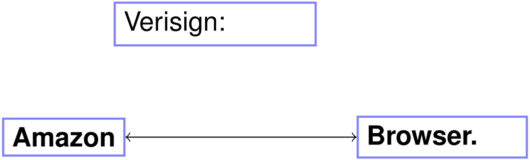

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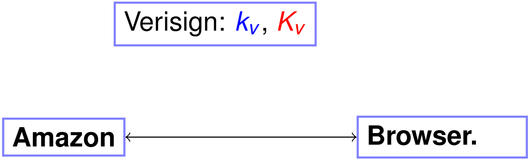

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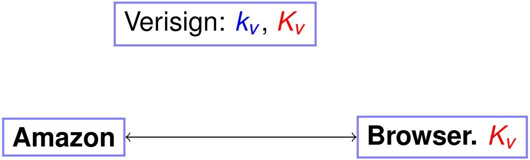

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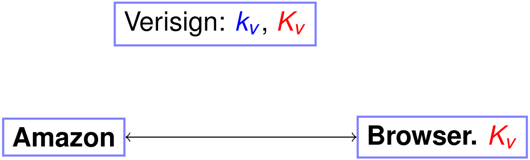

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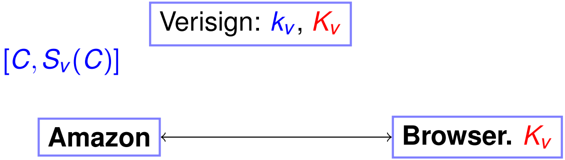

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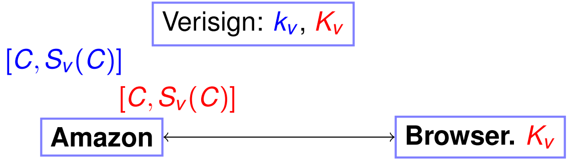

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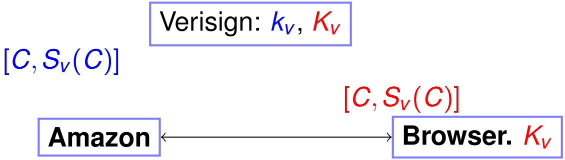

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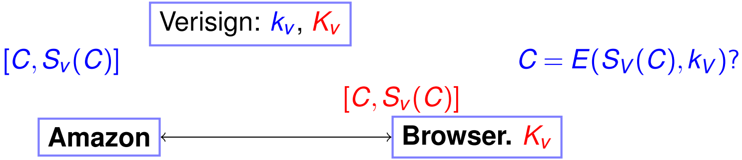
Signatures using RSA.



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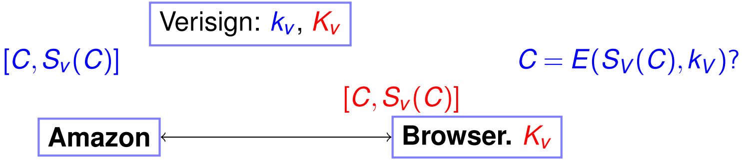

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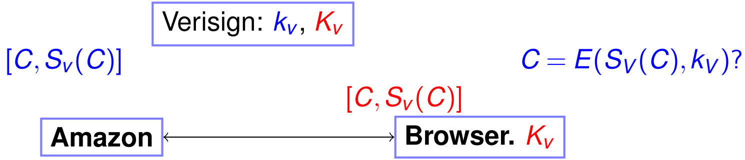

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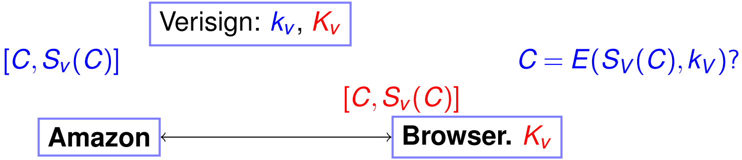

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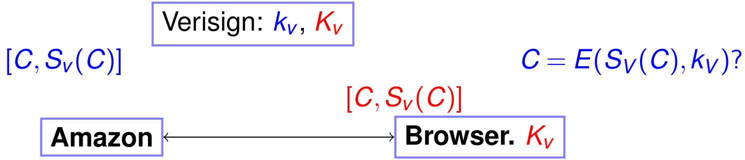

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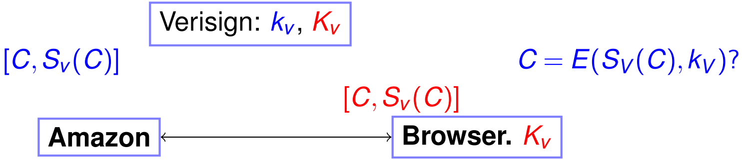

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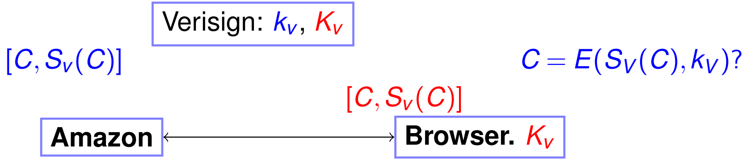

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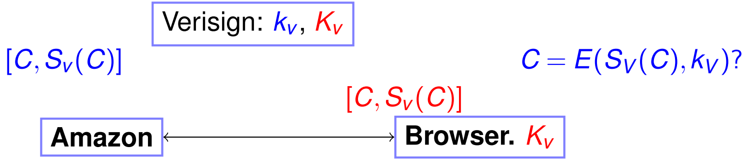

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Security: Eve can't forge unless she "breaks" RSA scheme.

RSA

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