Lecture 5: Graphs.

Graphs!

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Graphs!
Euler

Lecture 5: Graphs.

Graphs!
- Euler
- Definitions: model.

Lecture 5: Graphs.

Graphs!
- Euler
- Definitions: model.
- Fact!

Lecture 5: Graphs.

Graphs!
- Euler
- Definitions: model.
- Fact!
- Euler Again!!

Lecture 5: Graphs.

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Lecture 5: Graphs.

- Graphs!
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- Planar graphs.

Lecture 5: Graphs.

Graphs!
- Euler
- Definitions: model.
- Fact!
- Euler Again!!

Planar graphs.
- Euler Again!!!!

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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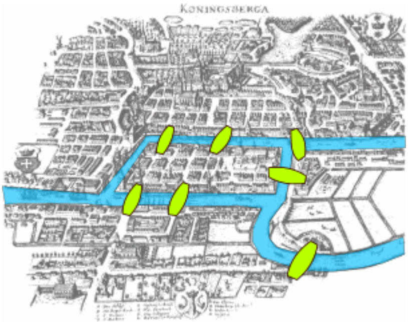

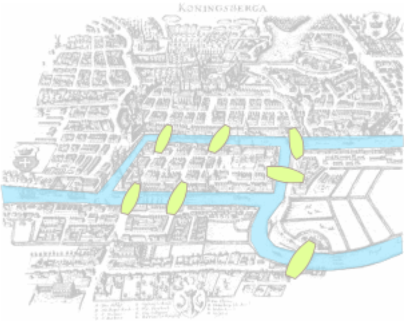

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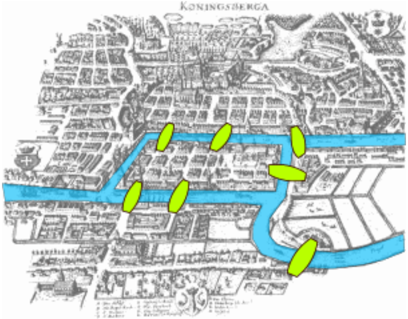

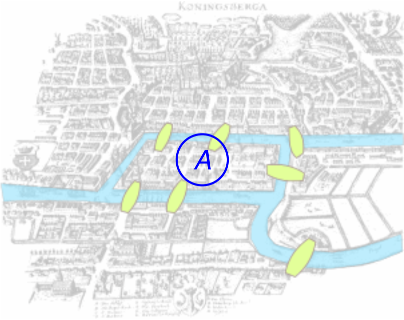

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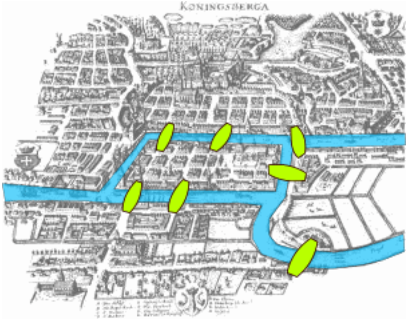

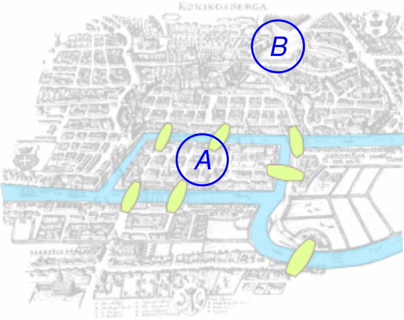

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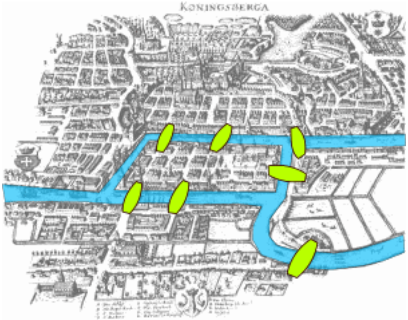

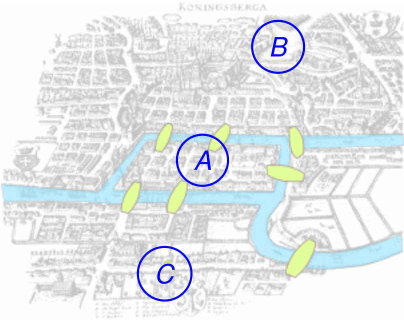

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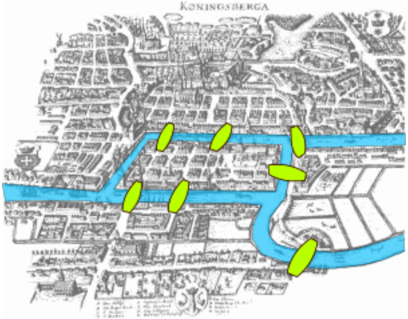

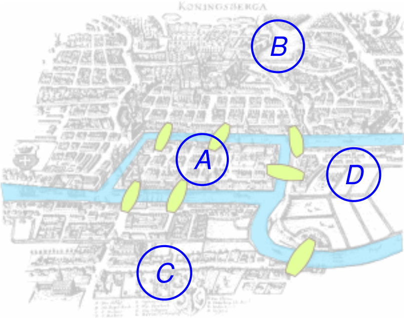

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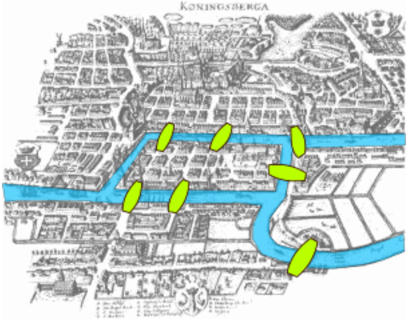

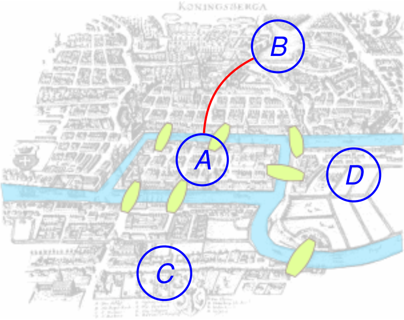

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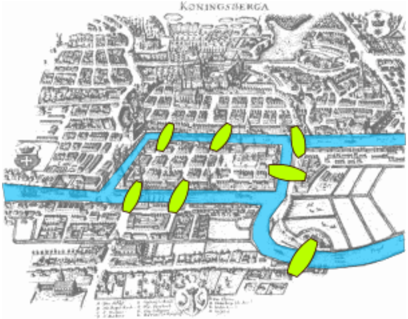

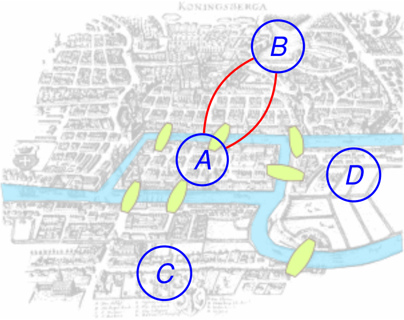

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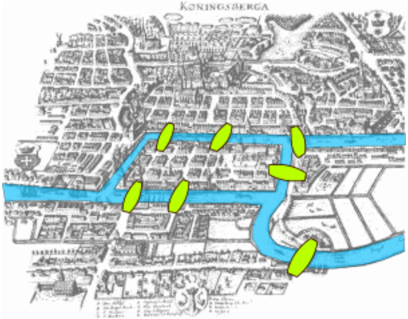

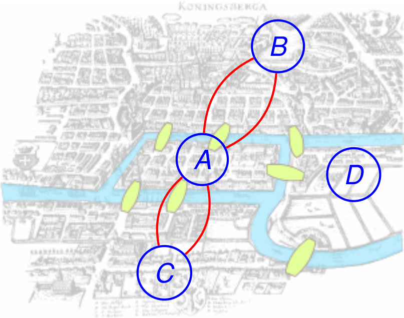

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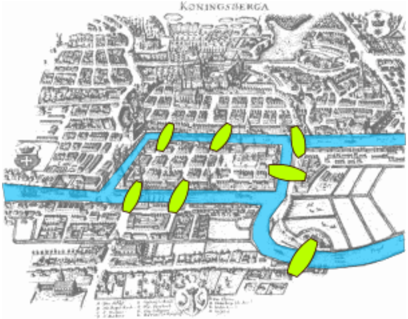

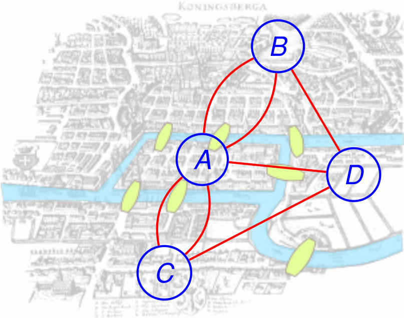

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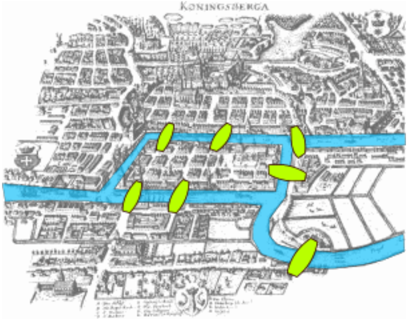

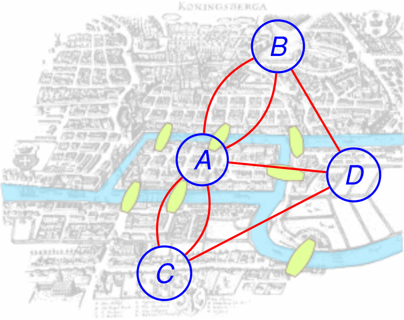

Can you draw a tour in the graph where you visit each edge once?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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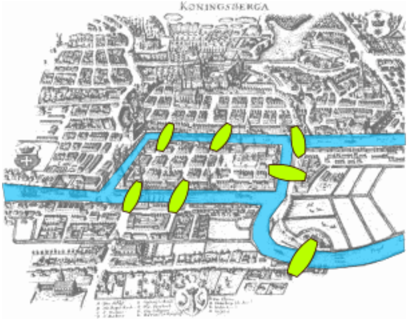

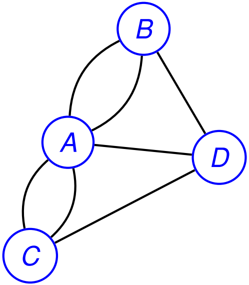

Can you draw a tour in the graph where you visit each edge once?
Yes?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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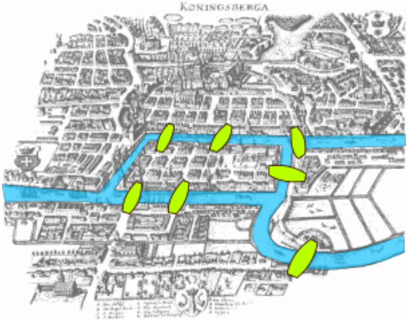

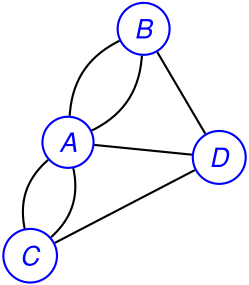

Can you draw a tour in the graph where you visit each edge once?
Yes? No?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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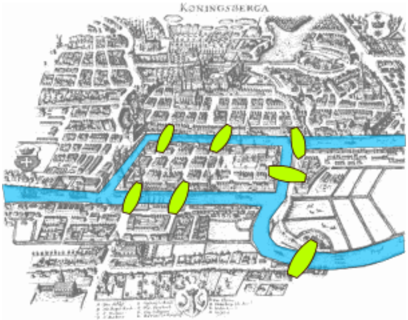
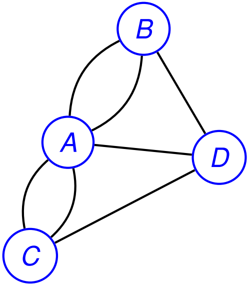
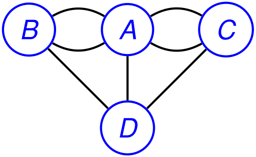




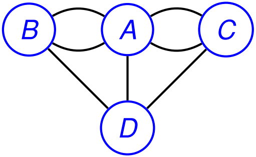
Can you draw a tour in the graph where you visit each edge once?
Yes? No?
We will see!

Graphs: formally.



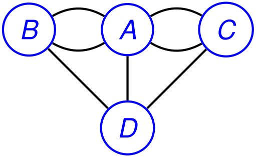
Graph:

Graphs: formally.



Graph: $G = (V, E)$.

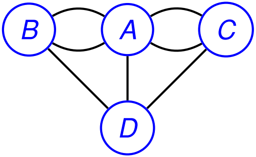
Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

Graphs: formally.

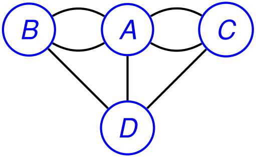


Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

Graphs: formally.



Graph: $G = (V, E)$.

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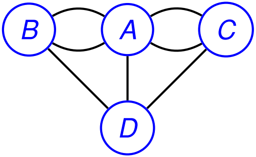
$E \subseteq V \times V$ -

Graphs: formally.



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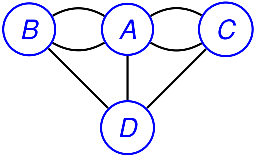


Graph: $G = (V, E)$.

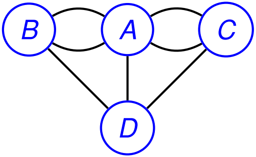
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

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$\{\{A, B\}, \{A, B\}$

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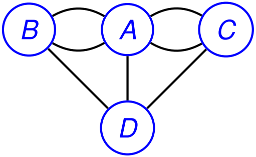

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$\{\{A, B\}, \{A, B\}, \{A, C\},$

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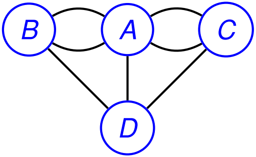

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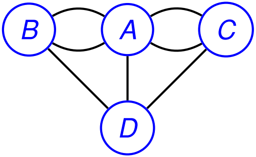
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

Graphs: formally.



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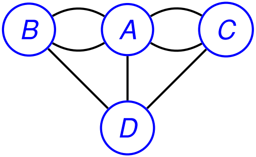
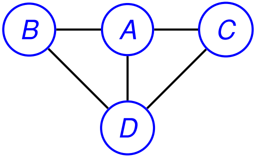
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

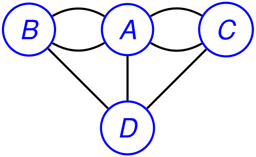
$E \subseteq V \times V$ - set of edges.

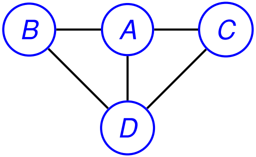
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{B, C\}, \{B, D\}, \{B, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

Graphs: formally.





Graph: $G = (V, E)$.

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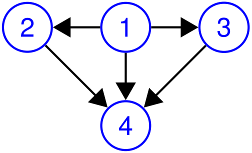
For CS 70, usually simple graphs.

No parallel edges.

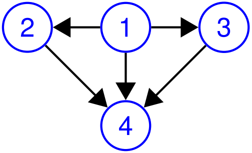
Multigraph above.

Directed Graphs



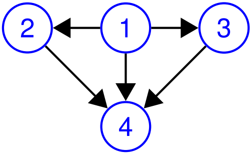
$G = (V, E)$.

Directed Graphs



$G = (V, E)$.
V - set of vertices.

Directed Graphs

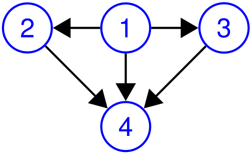


$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

Directed Graphs



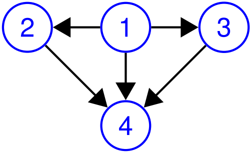
$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

Directed Graphs



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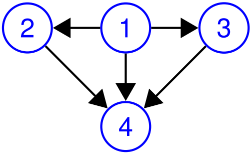
E ordered pairs of vertices.

$\{(1, 2),$

Directed Graphs



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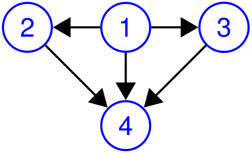
E ordered pairs of vertices.

$\{(1, 2), (1, 3),$

Directed Graphs



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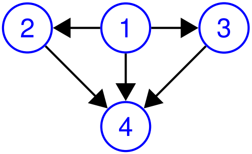
Directed Graphs



$G = (V, E)$.

V - set of vertices.

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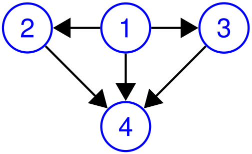
E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

Directed Graphs



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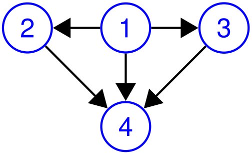
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.

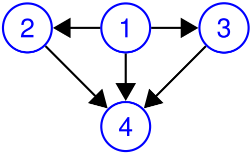
Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament:

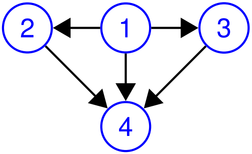
Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2,

Directed Graphs



$G = (V,E)$.
V - set of vertices.
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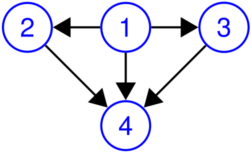
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence:

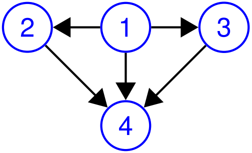
Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2,

Directed Graphs



$G = (V,E)$.
V - set of vertices.
$\{1,2,3,4\}$
E ordered pairs of vertices.
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One way streets.
Tournament: 1 beats 2, ...
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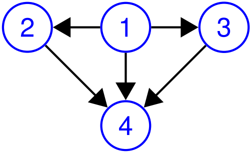
Directed Graphs



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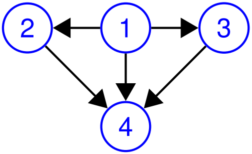
Social Network:

Directed Graphs



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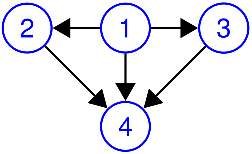

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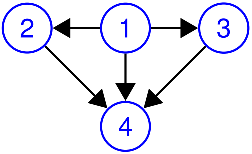
Social Network: Directed? Undirected?

Directed Graphs



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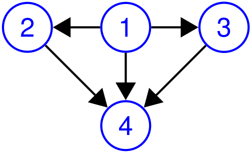
Social Network: Directed? Undirected?
Friends.

Directed Graphs



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Social Network: Directed? Undirected?
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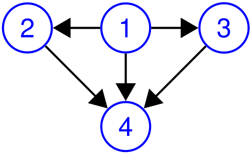
Directed Graphs



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Social Network: Directed? Undirected?
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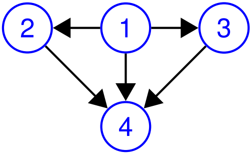
Likes.

Directed Graphs



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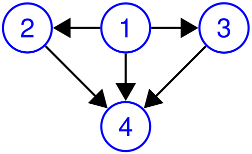
Likes. Directed.

Directed Graphs



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Graph Concepts and Definitions.

Graph: $G = (V, E)$

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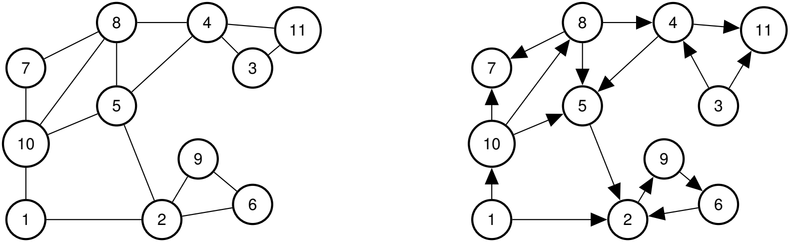

Neighbors of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

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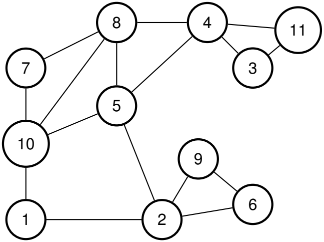

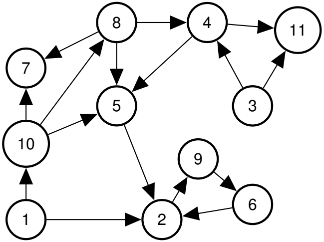

Neighbors of 10? 1,

Graph Concepts and Definitions.

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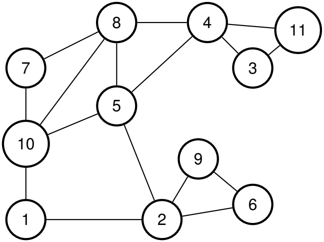

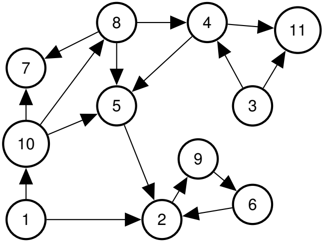

Neighbors of 10? 1,5,

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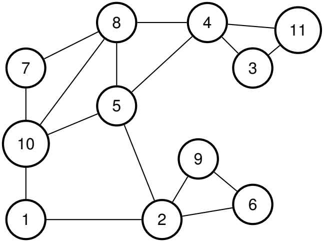

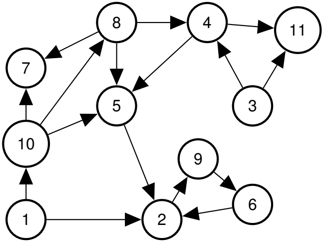

Neighbors of 10? 1,5,7,

Graph Concepts and Definitions.

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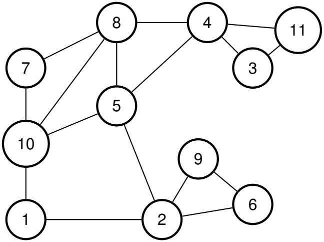

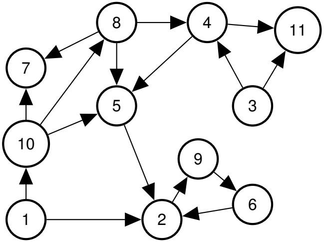

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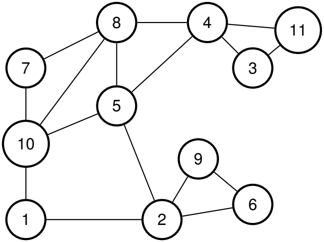

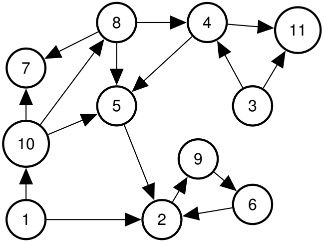

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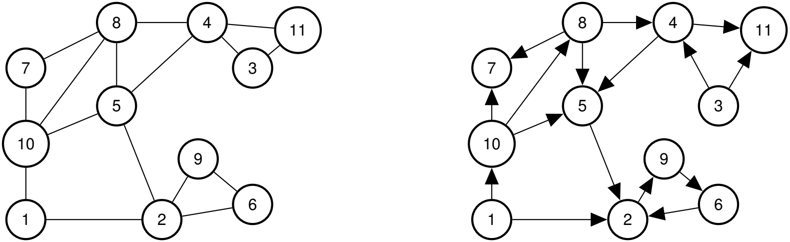

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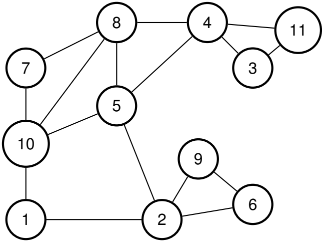

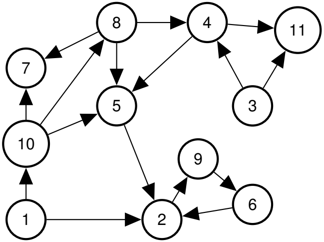

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Degree of vertex 1?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

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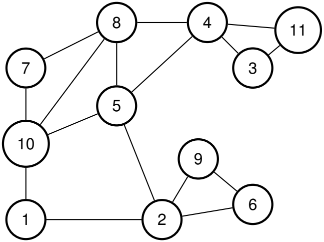

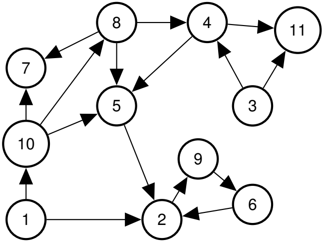

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Degree of vertex 1? 2

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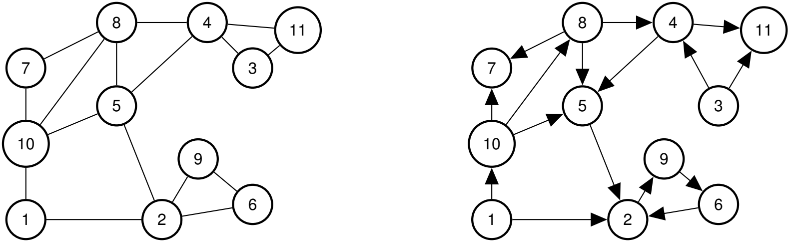

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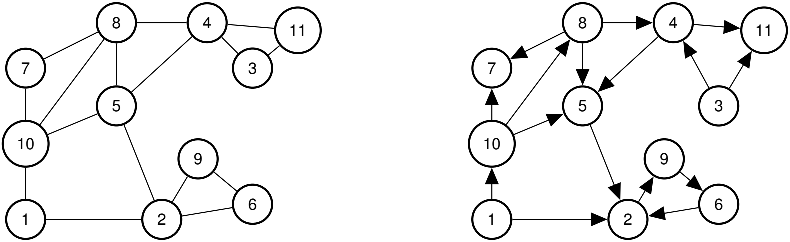

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Equals number of neighbors in simple graph.

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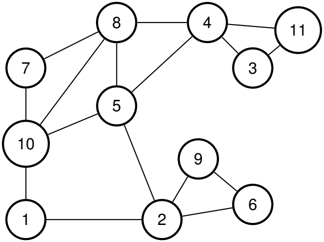

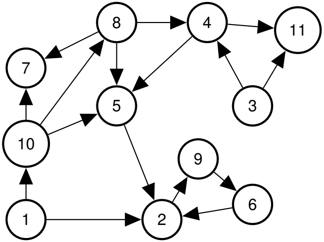

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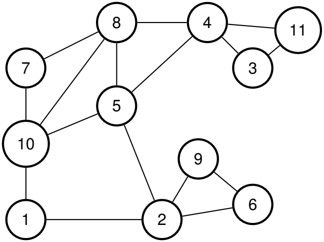

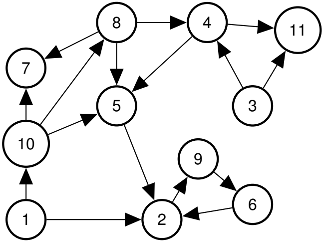

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Directed graph?

Graph Concepts and Definitions.

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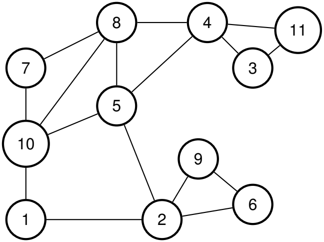
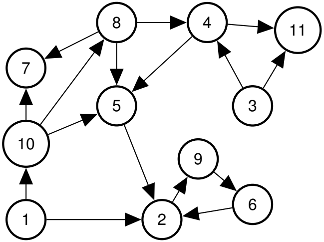
neighbors, adjacent, degree, incident, in-degree, out-degree





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Directed graph?

In-degree of 10?

Graph Concepts and Definitions.

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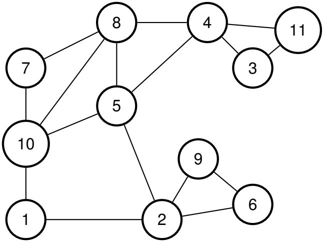
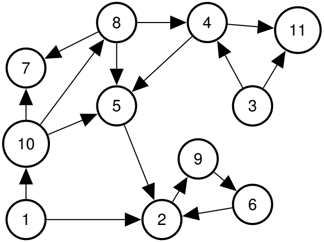
neighbors, adjacent, degree, incident, in-degree, out-degree





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Directed graph?

In-degree of 10? 1

Graph Concepts and Definitions.

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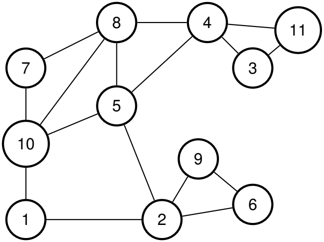

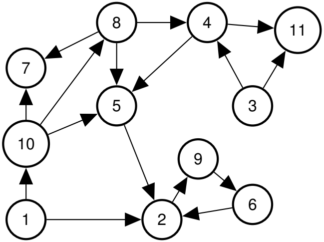

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In-degree of 10? 1   Out-degree of 10?

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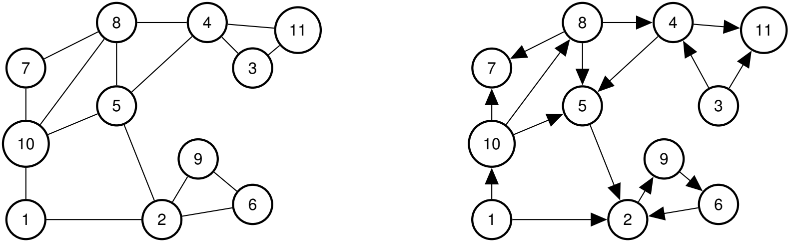

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Directed graph?

In-degree of 10? 1   Out-degree of 10? 3

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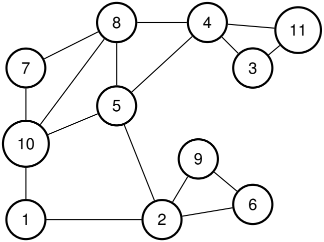

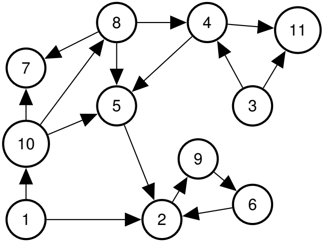

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Not (A)!

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Not (A)! Triangle.

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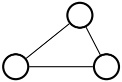

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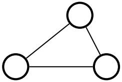

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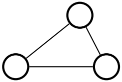

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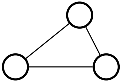


Not (B)! Triangle.

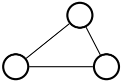
What?

Quick Proof.

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

Not (A)! Triangle.



Not (B)! Triangle.

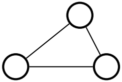
What? For triangle number of edges is 3, the sum of degrees is 6.

Quick Proof.

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

Not (A)! Triangle.



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What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...

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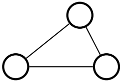

Not (B)! Triangle.

What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$? ..

Quick Proof.

The sum of the vertex degrees is equal to

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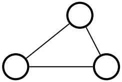

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What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$? ..or $2|V|$?

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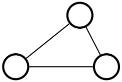

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What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$? ..or $2|V|$?

How many incidences does each edge contribute?

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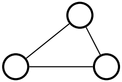

Not (B)! Triangle.

What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$? ..or $2|V|$?

How many incidences does each edge contribute? 2.

Quick Proof.

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(C) What?

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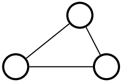

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Could it always be...$2|E|$? ..or $2|V|$?

How many incidences does each edge contribute? 2.
$2|E|$ incidences are contributed in total!

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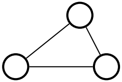

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What is degree v?

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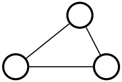

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How many incidences does each edge contribute? 2.
$2|E|$ incidences are contributed in total!
What is degree v? incidences contributed to v!

Quick Proof.

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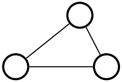

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sum of degrees is total incidences

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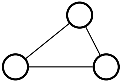

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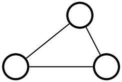

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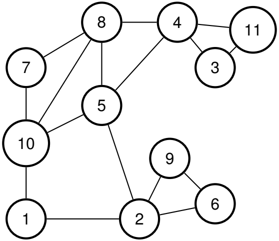
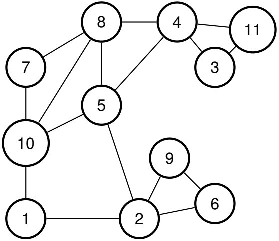
Thm: Sum of vertex degress is $2|E|$.

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Paths, walks, cycles, tour.



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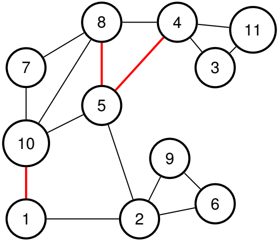
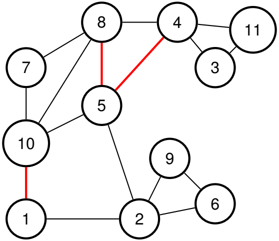


A path in a graph is a sequence of edges.

Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$?

Paths, walks, cycles, tour.



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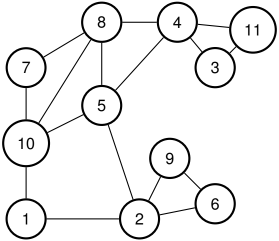

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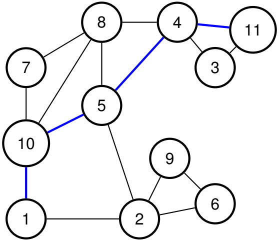

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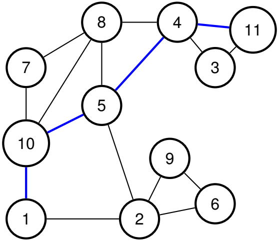
Paths, walks, cycles, tour.



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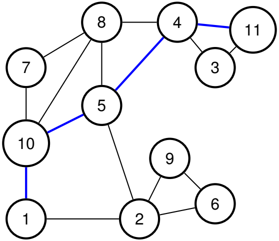

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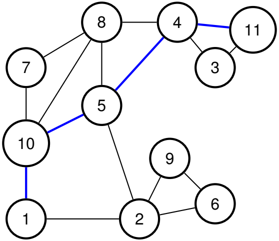
Path: $(v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)$.

Paths, walks, cycles, tour.



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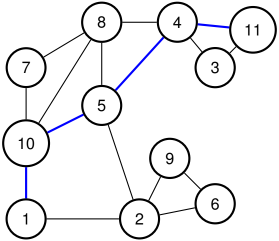
Quick Check!

Paths, walks, cycles, tour.



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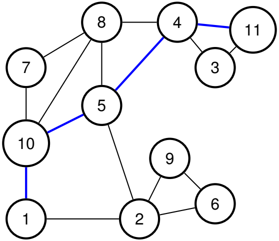
Quick Check! Length of path?

Paths, walks, cycles, tour.



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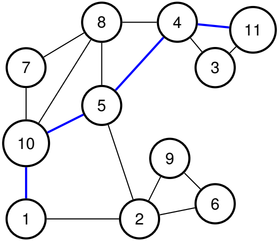
Quick Check! Length of path? k vertices

Paths, walks, cycles, tour.



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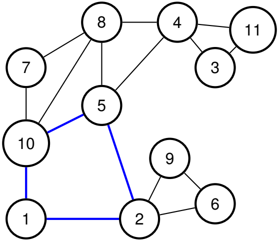
Quick Check! Length of path? k vertices or $k-1$ edges.

Paths, walks, cycles, tour.



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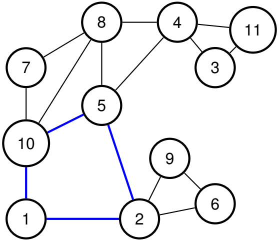
Quick Check! Length of path? k vertices or $k-1$ edges.

Cycle: Path with $v_1 = v_k$.

Paths, walks, cycles, tour.



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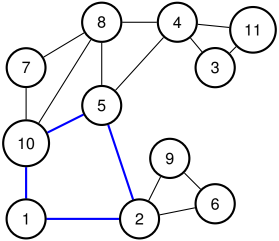
Cycle: Path with $v_1 = v_k$. Length of cycle?

Paths, walks, cycles, tour.



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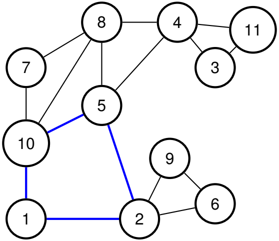
Quick Check! Length of path? k vertices or $k-1$ edges.
Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!

Paths, walks, cycles, tour.



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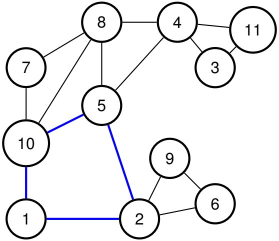
Quick Check! Length of path? k vertices or $k-1$ edges.
Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!
Path is usually simple.

Paths, walks, cycles, tour.



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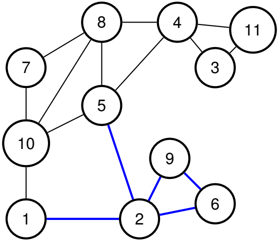
Paths, walks, cycles, tour.



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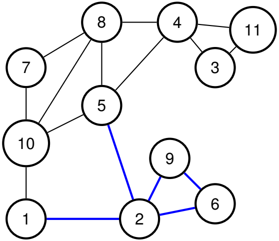
Walk is sequence of edges with possible repeated vertex or edge.

Paths, walks, cycles, tour.



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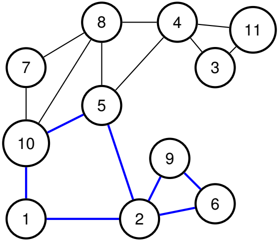

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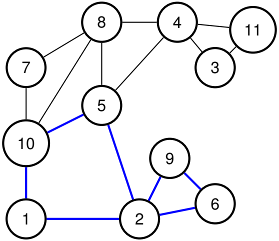
Paths, walks, cycles, tour.



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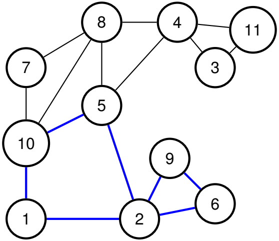
Paths, walks, cycles, tour.



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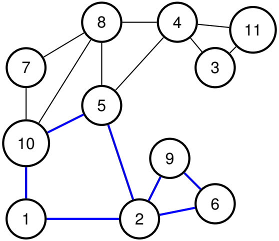
Paths, walks, cycles, tour.



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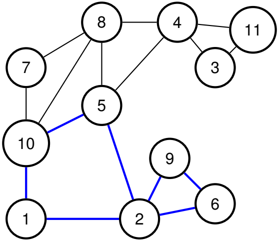
Tour is walk that starts and ends at the same node.

Quick Check!
Path is to Walk as Cycle is to ??

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1,10\}, \{8,5\}, \{4,5\}$? No!
Path? $\{1,10\}, \{10,5\}, \{5,4\}, \{4,11\}$? Yes!

Path: $(v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)$.
Quick Check! Length of path? k vertices or $k-1$ edges.
Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!
Path is usually simple. No repeated vertex!
Walk is sequence of edges with possible repeated vertex or edge.
Tour is walk that starts and ends at the same node.

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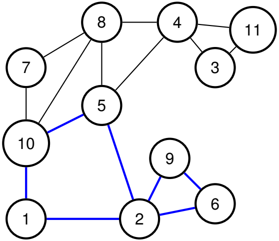

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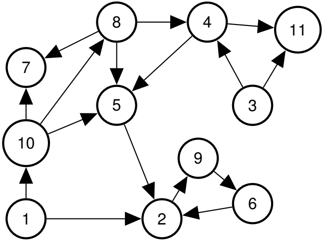

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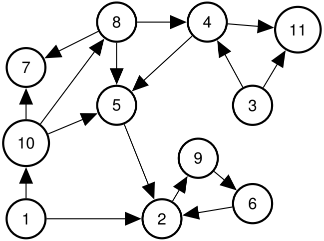

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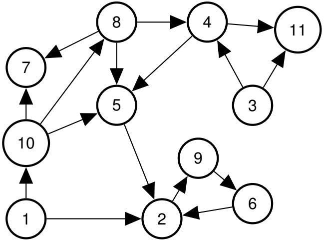

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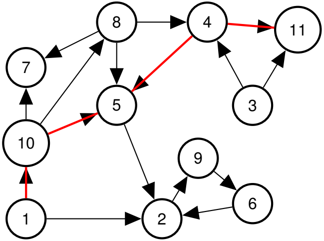

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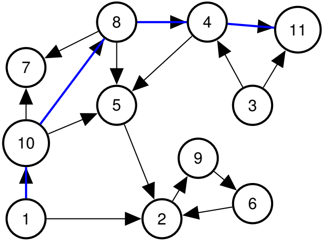

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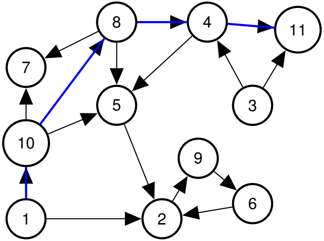

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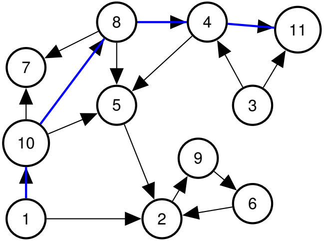

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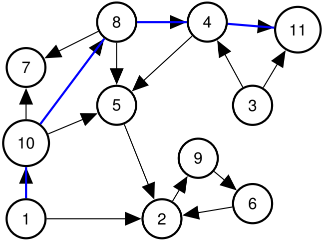

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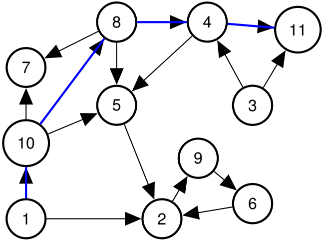

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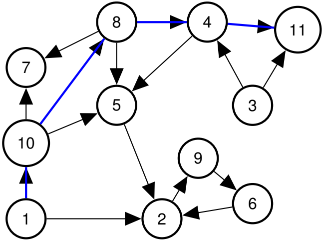

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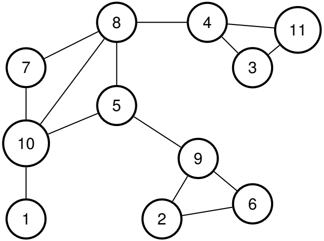
Paths, walks, cycles, tours ... are analagous to undirected now.

Connectivity: undirected graph.



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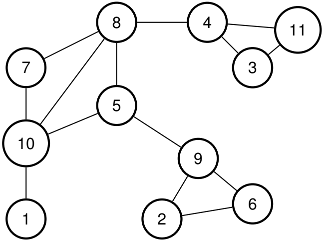

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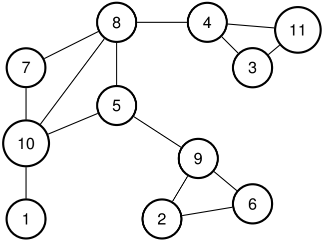

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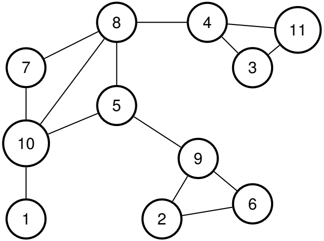

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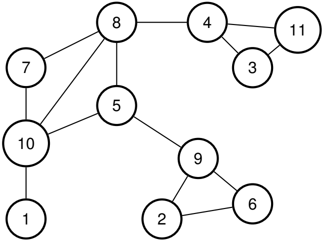

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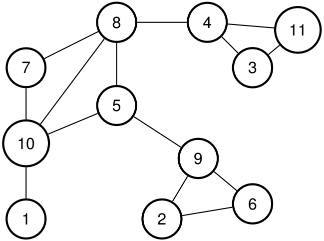

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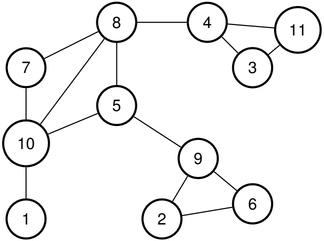

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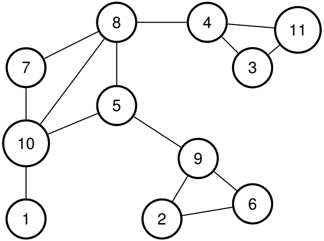

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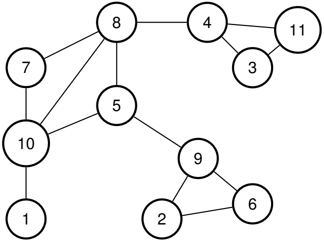

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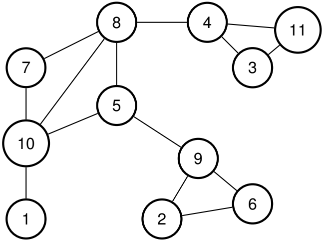

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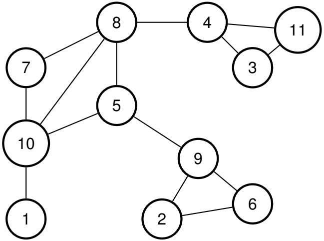

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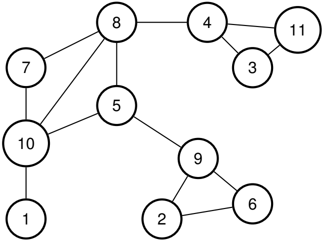

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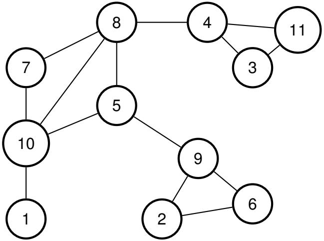

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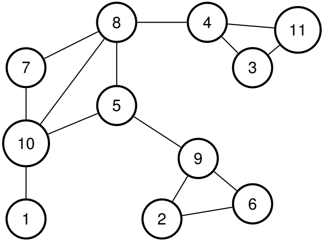

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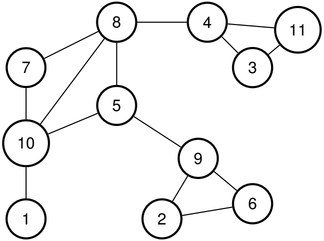

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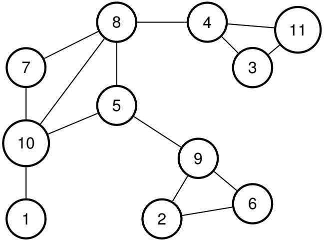

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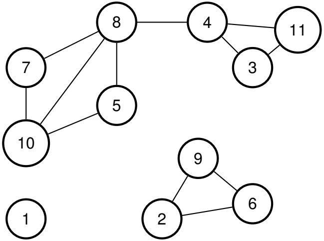

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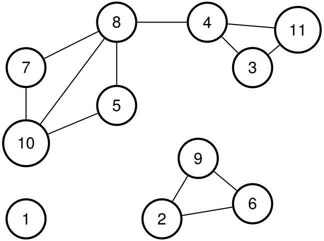
How about now?



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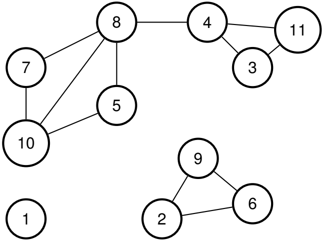

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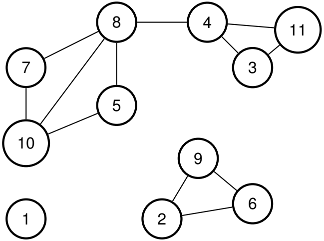
Connected Components?



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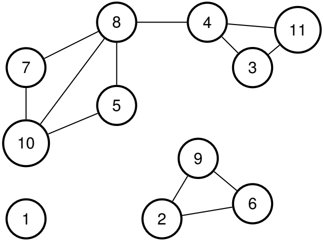
Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.



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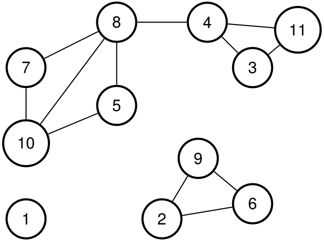

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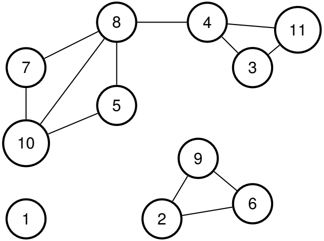

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Finally..back to Euler!

An Eulerian Tour is a tour that visits each edge exactly once.

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Tour enters and leaves vertex v on each visit.

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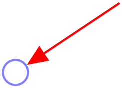

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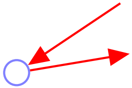

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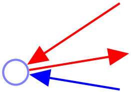

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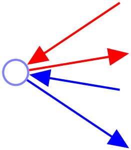

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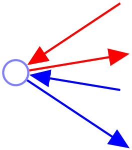

When you enter, you can leave.
For starting node,

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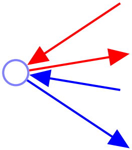

When you enter, you can leave.
For starting node, tour leaves first

Finally..back to Euler!

An Eulerian Tour is a tour that visits each edge exactly once.

Theorem: Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.
Proof of only if: Eulerian $\Longrightarrow$ connected and all even degree.

Eulerian Tour is connected so graph is connected.
Tour enters and leaves vertex v on each visit.
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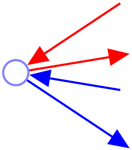

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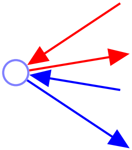

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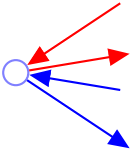

When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.
Not The Hotel California.

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm.

Finding a tour!

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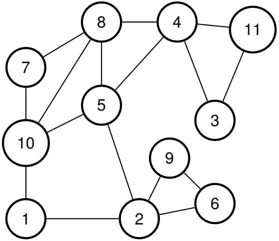

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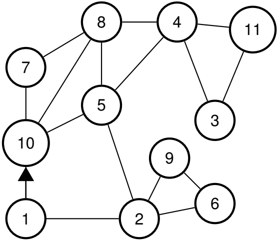

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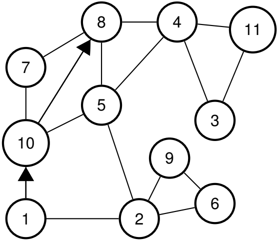

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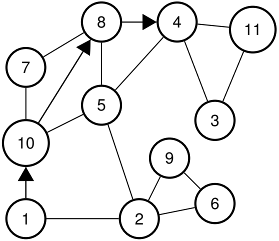

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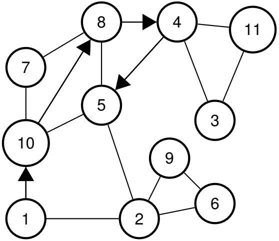

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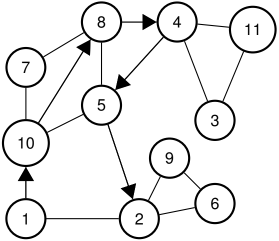

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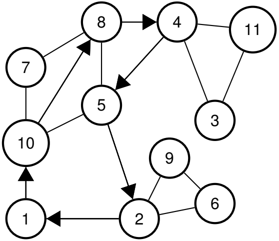

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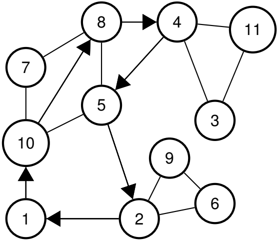

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2. Remove tour, C.

Finding a tour!

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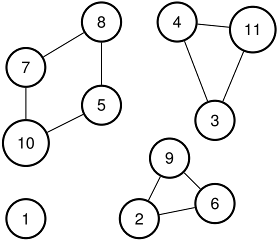

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Finding a tour!

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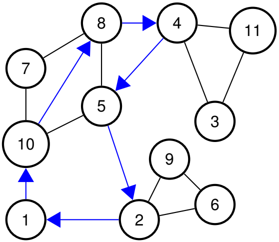

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Finding a tour!

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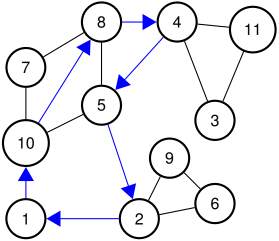

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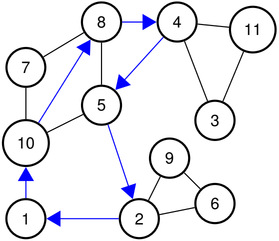

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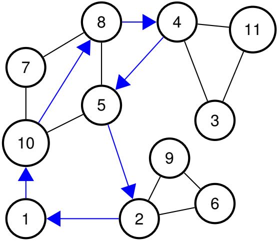

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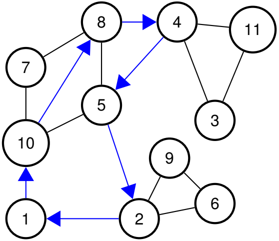

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    Example: $v_1 = 1$,

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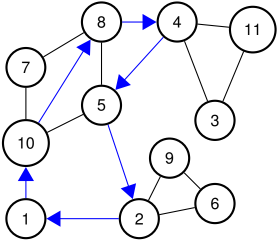

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 Example: $v_1 = 1$, $v_2 = 10$,

Finding a tour!

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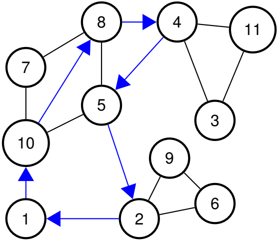

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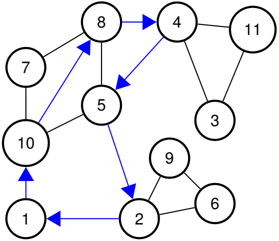

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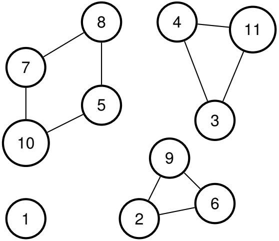

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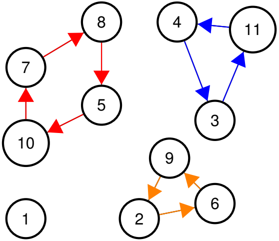

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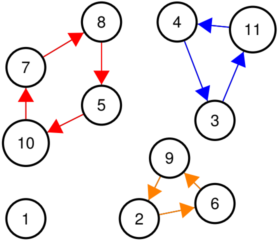

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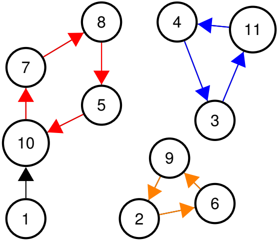

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   1,10

Finding a tour!

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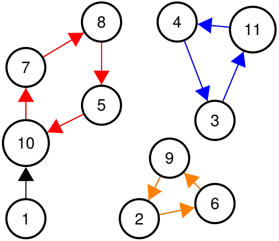

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1,10,7,8,5,10

Finding a tour!

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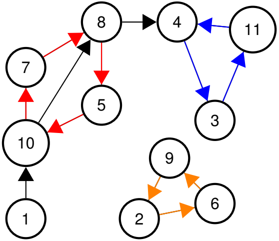

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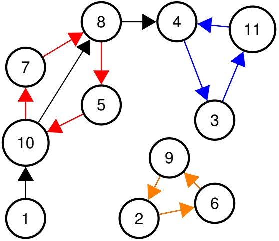

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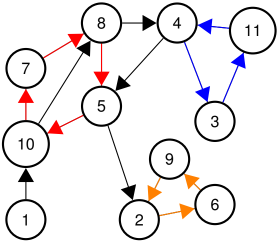

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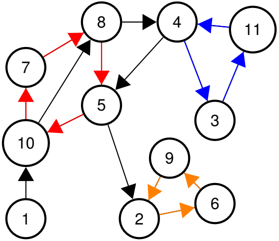

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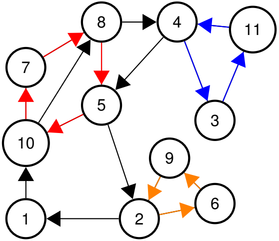

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   1,10,7,8,5,10 ,8,4,3,11,4 5,2,6,9,2 and to 1!

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

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Proof of Claim: Even degree.

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Proof of Claim: Even degree. If enter, can leave

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Recursive/Inductive Algorithm.

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Resulting graph may be disconnected. (Removed edges!)

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Let components be $G_1, \ldots, G_k$.

Recursive/Inductive Algorithm.

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Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.
Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.

Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)
Let components be $G_1, \ldots, G_k$.
Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?

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G was connected $\implies$

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a vertex in G_i must be incident to a removed edge in C.

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Claim: Each vertex in each G_i has even degree

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Claim: Each vertex in each G_i has even degree and is connected.

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Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v.

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Prf: Tour C has even incidences to any vertex v. □

3. Find tour T_i of G_i starting/ending at v_i.

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3. Find tour T_i of G_i starting/ending at v_i. Induction.

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Visits every edge once:
Visits edges in C

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Visits every edge once:
Visits edges in C exactly once.
By induction for all edges in each G_i.

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Break time!

Well admin time!

Break time!

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Must choose homework option or test only: soon after recieving hw 1 scores.

Break time!

Well admin time!

Must choose homework option or test only: soon after recieving hw 1 scores.

Test Option: don’t have to do homework.

Break time!

Well admin time!

Must choose homework option or test only: soon after recieving hw 1 scores.

Test Option: don't have to do homework. Yes!!

Break time!

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Must choose homework option or test only: soon after recieving hw 1 scores.

- Test Option: don't have to do homework. Yes!!
  - Should do homework.

Break time!

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Must choose homework option or test only: soon after recieving hw 1 scores.

Test Option: don't have to do homework. Yes!!

Should do homework. No need to write up.

Break time!

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Homework Option: have to do homework.

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The truth:

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The truth: mostly test,

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The truth: mostly test, both options!

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- Variance mostly in exams for A/B range.

Break time!

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The truth: mostly test, both options!
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How will I do?

Break time!

Well admin time!

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How will I do?

Mostly up to you.

Planar graphs.

A graph that can be drawn in the plane without edge crossings.

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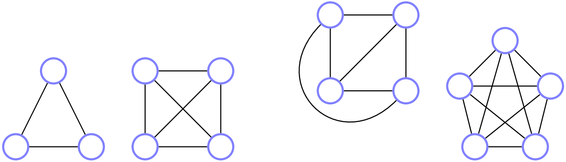

Planar?

Planar graphs.

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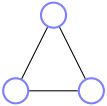
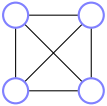
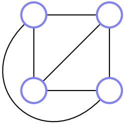
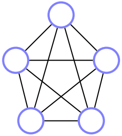

Planar? Yes for Triangle.

Planar graphs.

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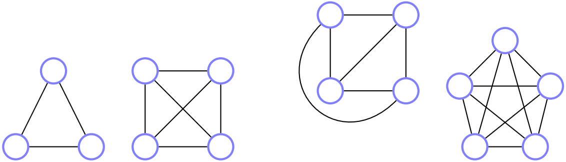

Planar? Yes for Triangle.
Four node complete?

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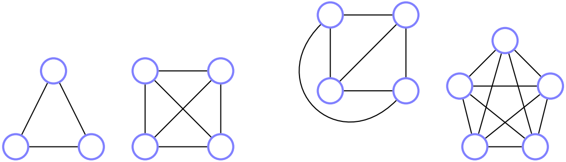

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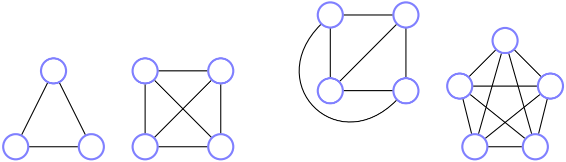

Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or K_5?

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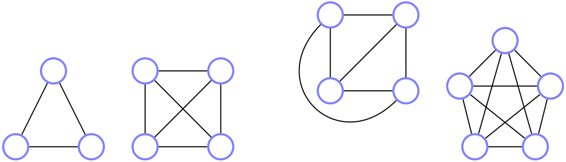

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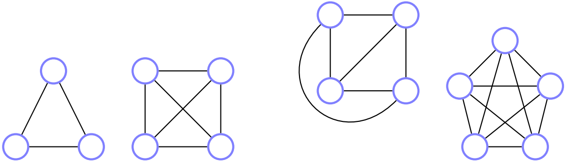

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Four node complete? Yes.
Five node complete or K_5? No! Why?

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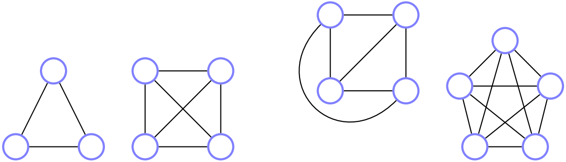

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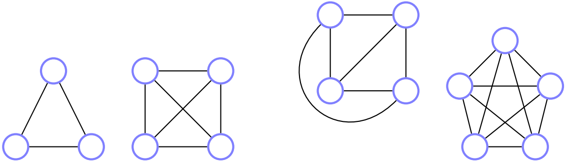

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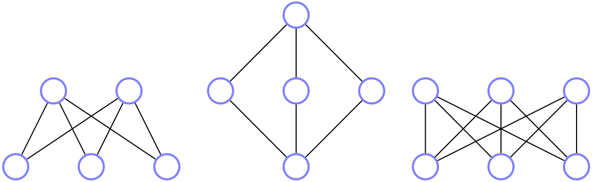

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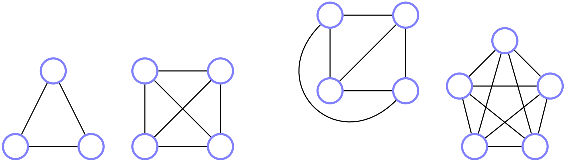

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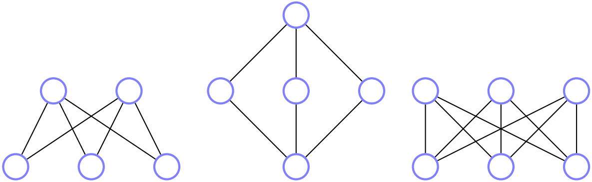

Two to three nodes, bipartite?

Planar graphs.

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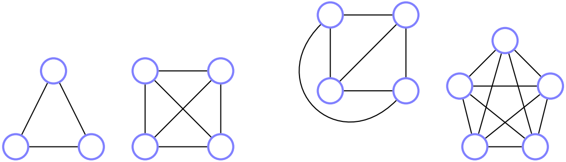

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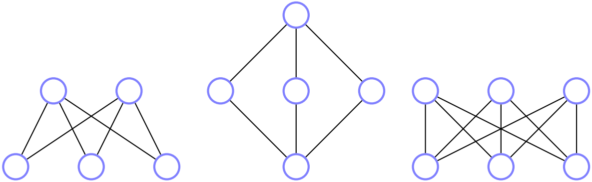

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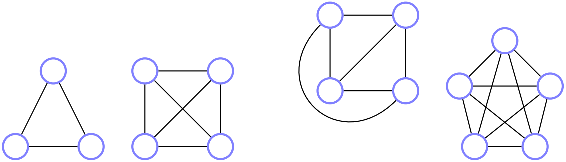

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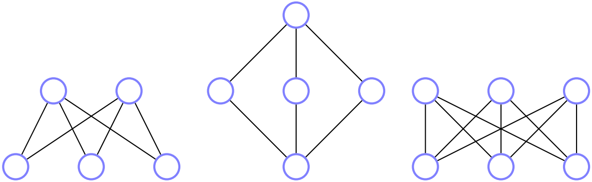

Two to three nodes, bipartite? Yes.
Three to three nodes, complete/bipartite or $K_{3,3}$.

Planar graphs.

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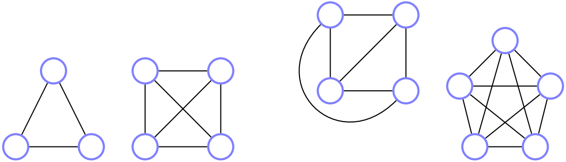

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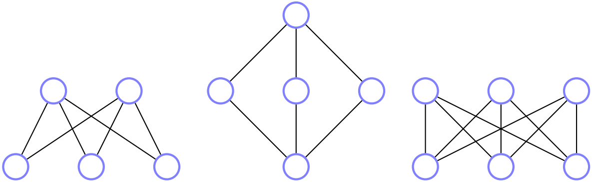

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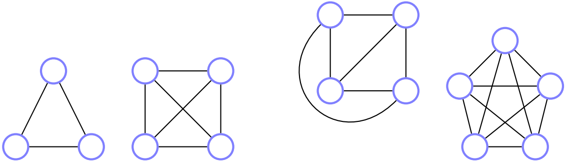

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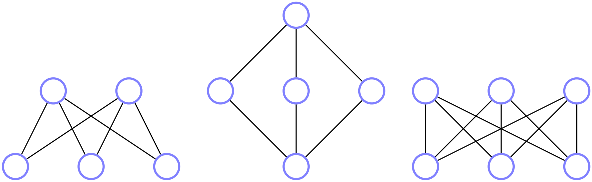

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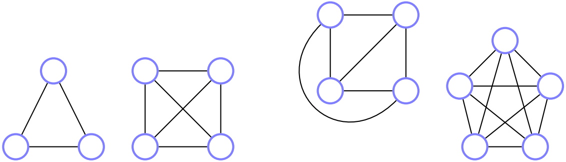

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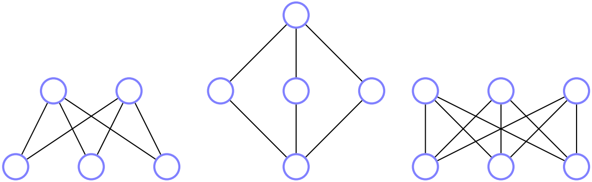

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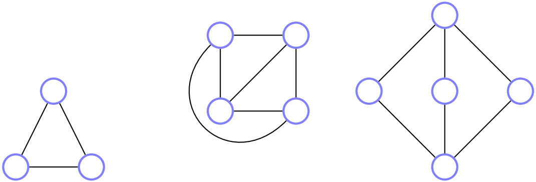
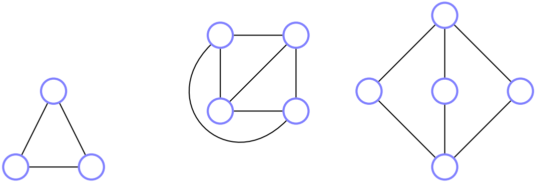
Euler's Formula.

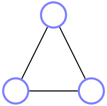
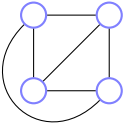
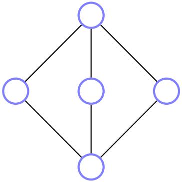


Euler's Formula.



Faces: connected regions of the plane.

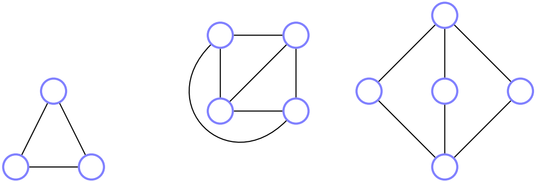
Euler's Formula.

Faces: connected regions of the plane.

How many faces for

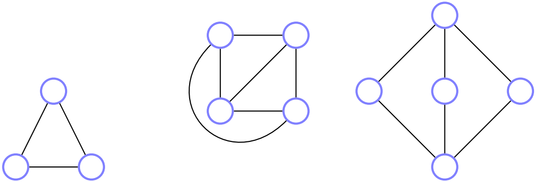
Euler's Formula.



Faces: connected regions of the plane.

How many faces for
triangle?

Euler's Formula.

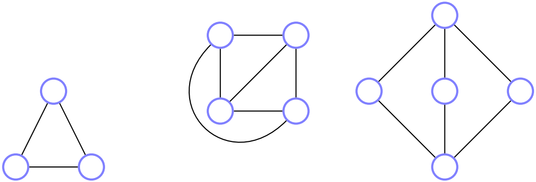


Faces: connected regions of the plane.

How many faces for

triangle? 2

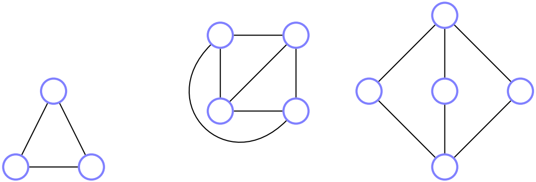
Euler's Formula.



Faces: connected regions of the plane.

How many faces for
  triangle? 2
  complete on four vertices or K_4?

Euler's Formula.



Faces: connected regions of the plane.

How many faces for

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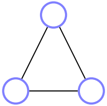
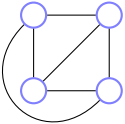
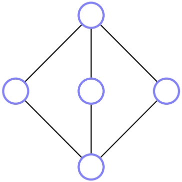
complete on four vertices or K_4? 4

Euler's Formula.

Faces: connected regions of the plane.

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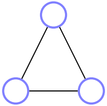
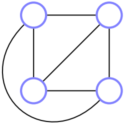
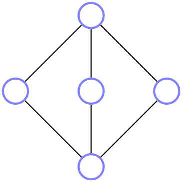
Euler's Formula.

Faces: connected regions of the plane.

How many faces for

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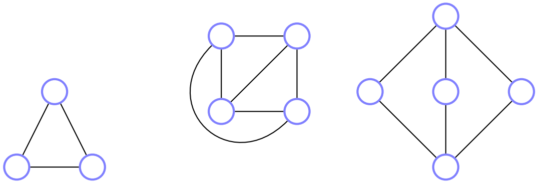

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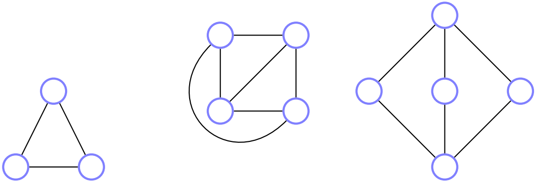

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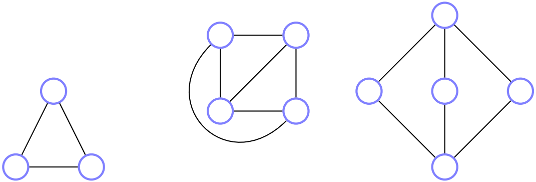

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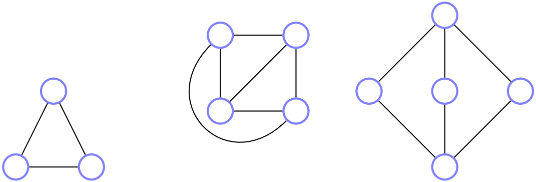

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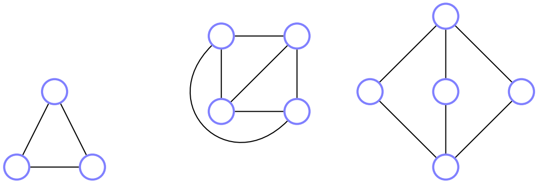

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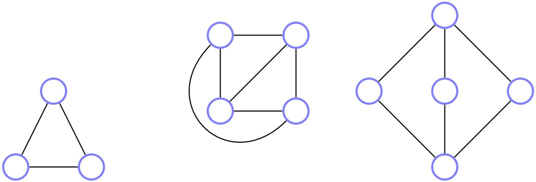

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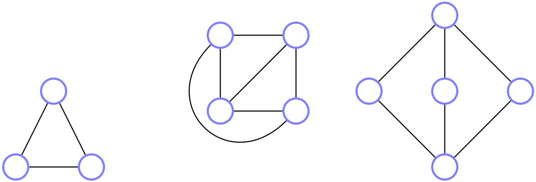

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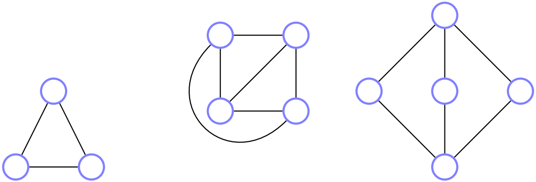

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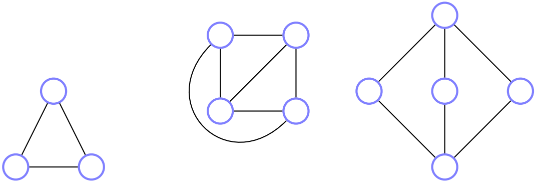
Euler's Formula.



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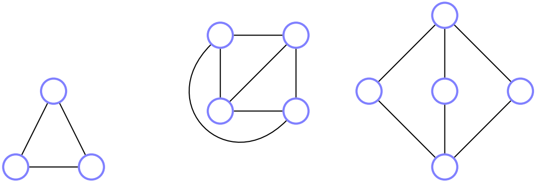

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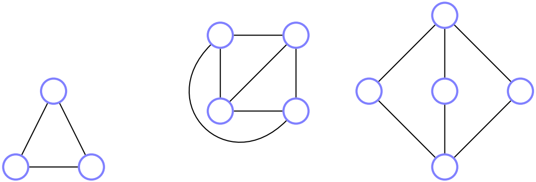

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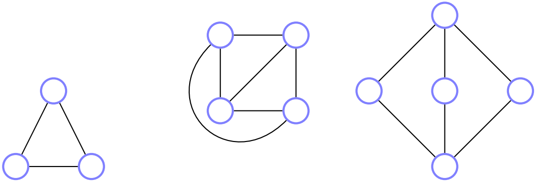

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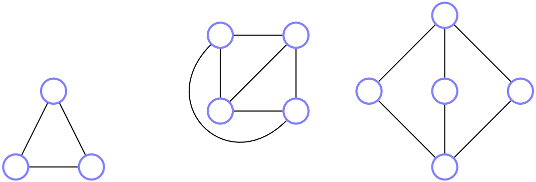
Examples = 3!

Euler's Formula.



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Examples = 3! Proven!

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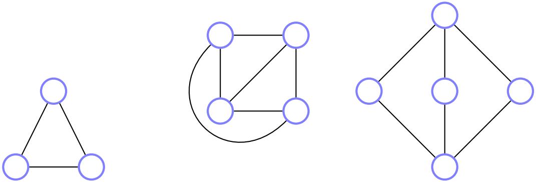

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Examples = 3! Proven! Not!!!!

Euler and Polyhedron.

Greeks knew formula for polyhedron.

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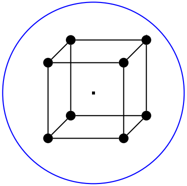

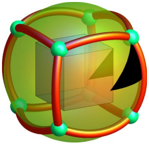

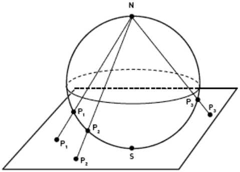

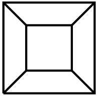

Euler and Polyhedron.

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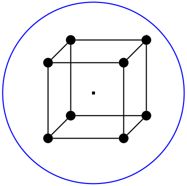

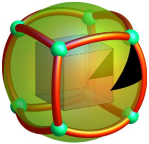

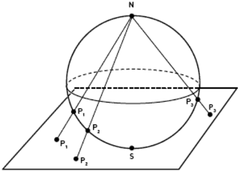

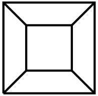

Faces?

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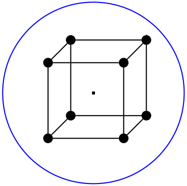

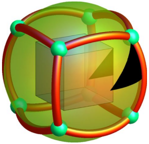

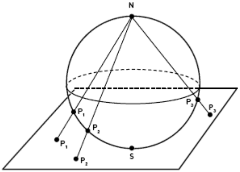

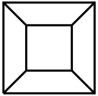

Faces? 6. Edges?

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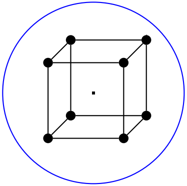

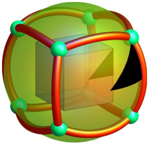

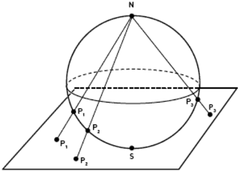

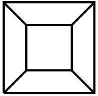

Faces? 6.   Edges? 12.

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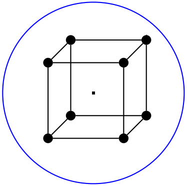

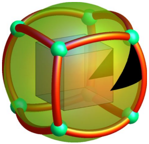

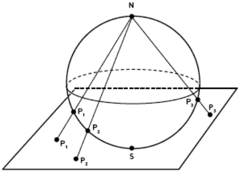

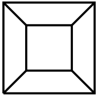

Faces? 6. Edges? 12. Vertices?

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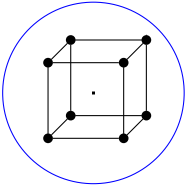

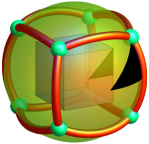

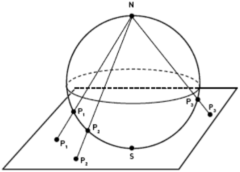

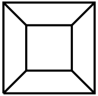

Faces? 6. Edges? 12. Vertices? 8.

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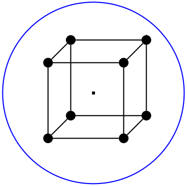

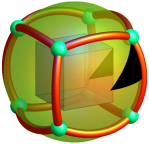

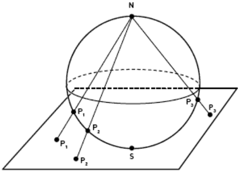

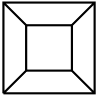

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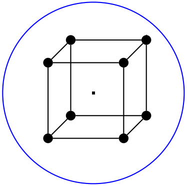

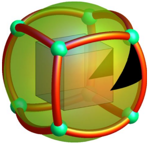

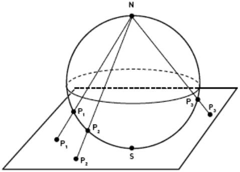

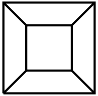

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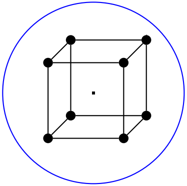

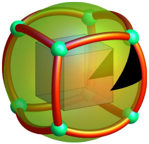

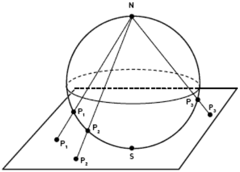

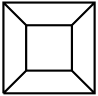

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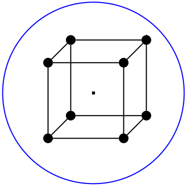

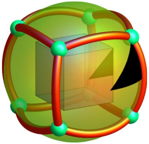

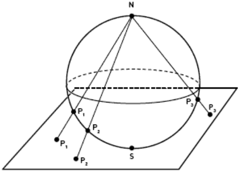

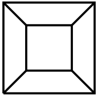

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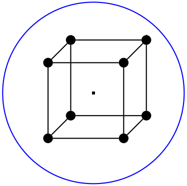

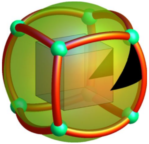

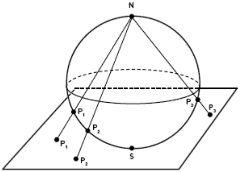

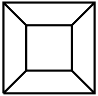

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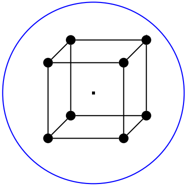
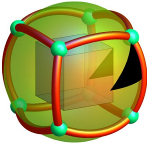
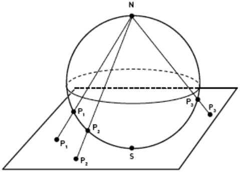

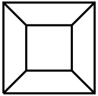

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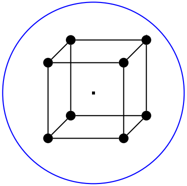
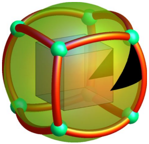
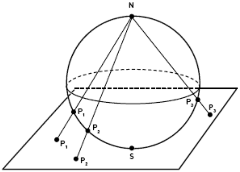

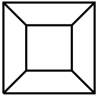

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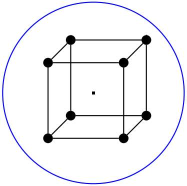

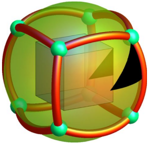

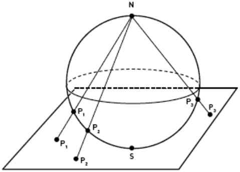

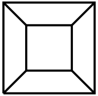

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Polyhedron without holes

Euler and Polyhedron.

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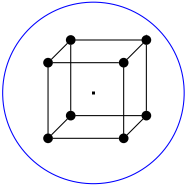
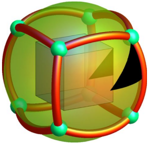
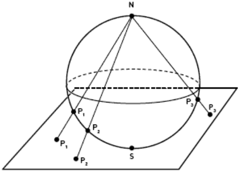

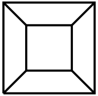

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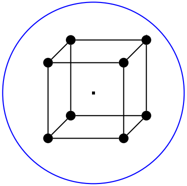

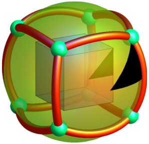

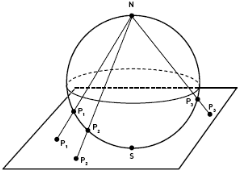

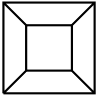

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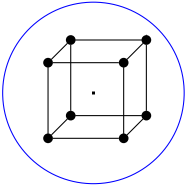

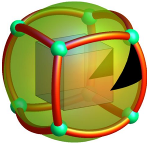

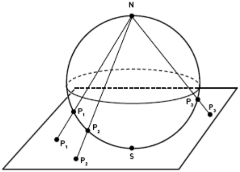

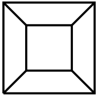

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Surround by sphere.

Euler and Polyhedron.

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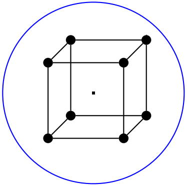
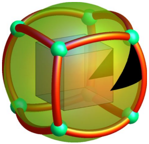
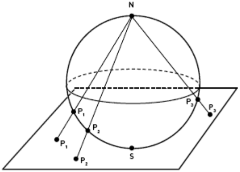

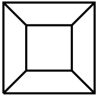

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Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.

Euler and Polyhedron.

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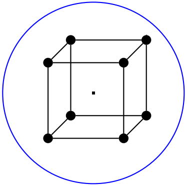

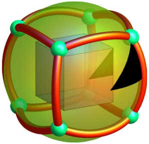

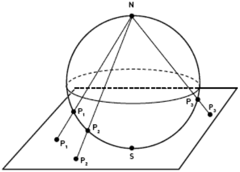

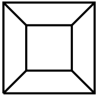

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Surround by sphere.
Project from point inside polytope onto sphere.
Sphere

Euler and Polyhedron.

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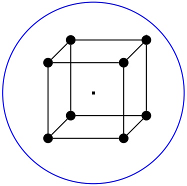
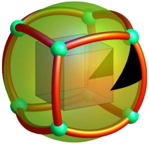
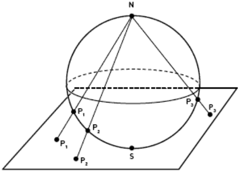

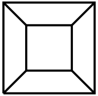

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Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.
Sphere $\equiv$ Plane!

Euler and Polyhedron.

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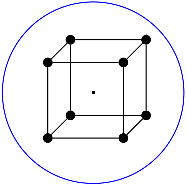
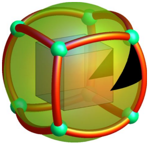
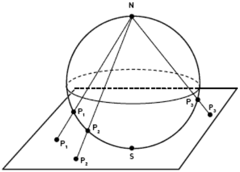

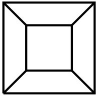

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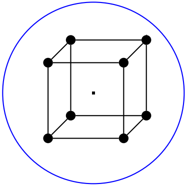
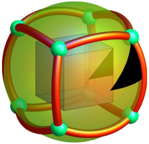
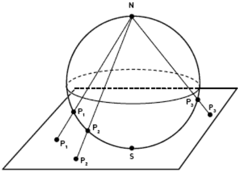

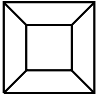

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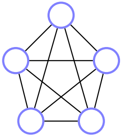
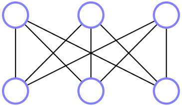
Surround by sphere.
Project from point inside polytope onto sphere.
Sphere $\equiv$ Plane! Topologically.

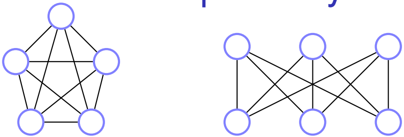
Euler proved formula thousands of years later!

Euler and non-planarity of K_5 and $K_{3,3}$





Euler and non-planarity of K_5 and $K_{3,3}$



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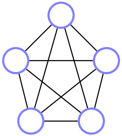

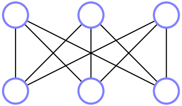

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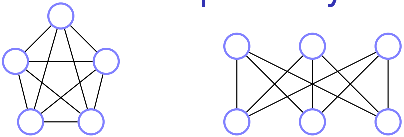

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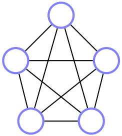

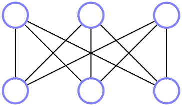

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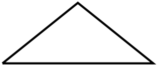

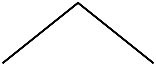

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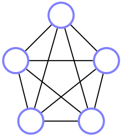

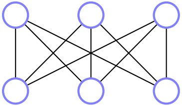

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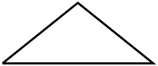

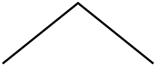

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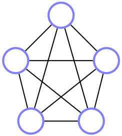

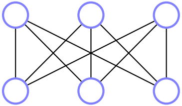

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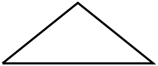

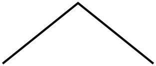

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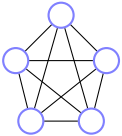
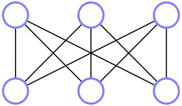
Euler and non-planarity of K_5 and $K_{3,3}$





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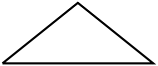

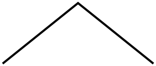

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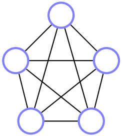

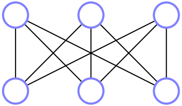

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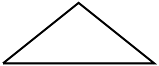

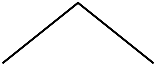

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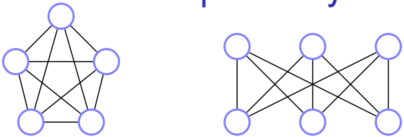

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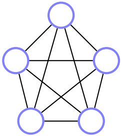

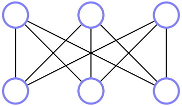

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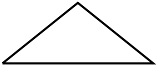

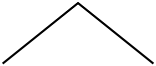

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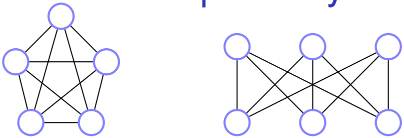

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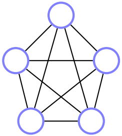

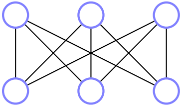

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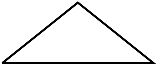

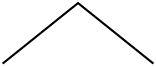

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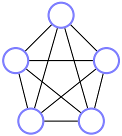

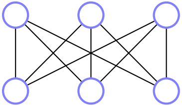

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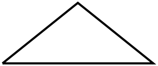

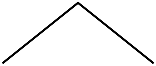

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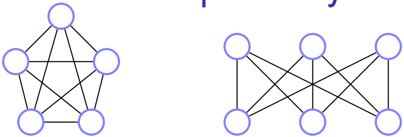

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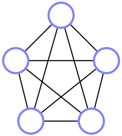

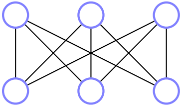

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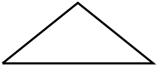

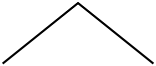

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Plug into Euler:

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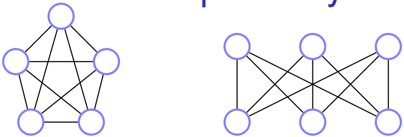

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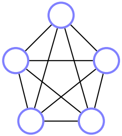
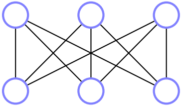

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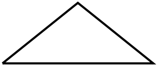
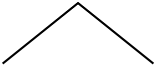

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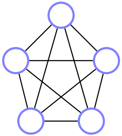

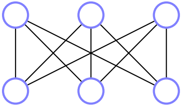

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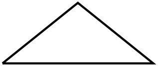

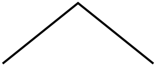

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K_5

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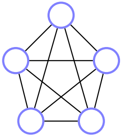
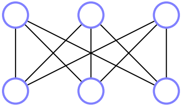

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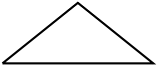
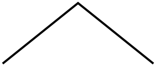

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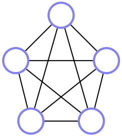
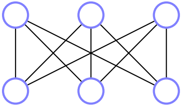
Plug into Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

K_5 Edges?

Euler and non-planarity of K_5 and $K_{3,3}$





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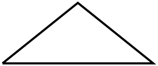

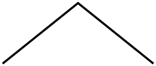

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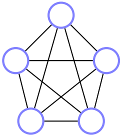

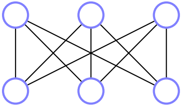

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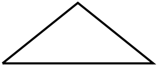

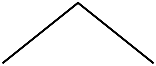

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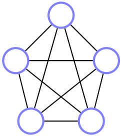
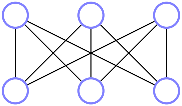
Euler and non-planarity of K_5 and $K_{3,3}$





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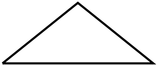

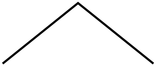

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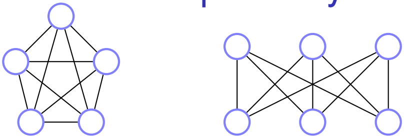
Euler and non-planarity of K_5 and $K_{3,3}$



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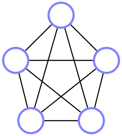
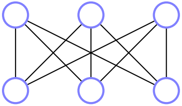
K_5 Edges? $4+3+2+1=10$. Vertices? 5.

Euler and non-planarity of K_5 and $K_{3,3}$





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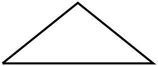

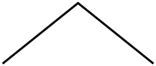

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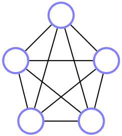
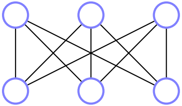
$10 \not\leq 3(5) - 6 = 9$.

Euler and non-planarity of K_5 and $K_{3,3}$





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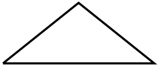

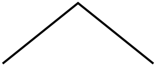

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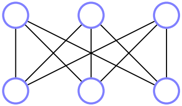
K_5 Edges? $4 + 3 + 2 + 1 = 10$. Vertices? 5.

$10 \not\leq 3(5) - 6 = 9$. $\implies$ K_5 is not planar.

Proving non-planarity for $K_{3,3}$



Euler's formula $\implies$ $3f \leq 2e$

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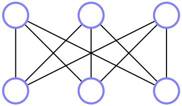
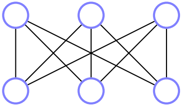


Euler's formula $\implies$ $3f \leq 2e$ for any planar graph.

Proving non-planarity for $K_{3,3}$



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$K_{3,3}$?

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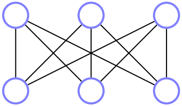

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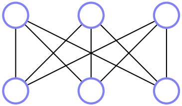

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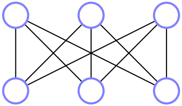

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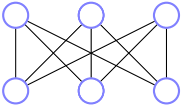
$K_{3,3}$? Edges? 9. Vertices. 6.

Proving non-planarity for $K_{3,3}$



Euler's formula $\implies$ $3f \leq 2e$ for any planar graph.

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$$9 \leq 3(6) - 6?$$

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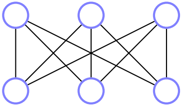

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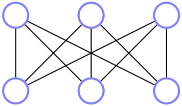

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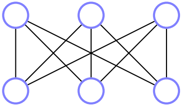

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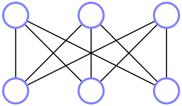

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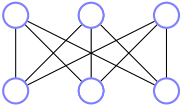

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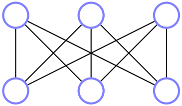

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But no cycles that are triangles. Face is of length ≥ 4.

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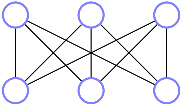

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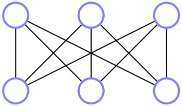

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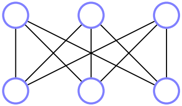

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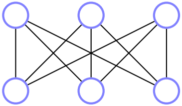

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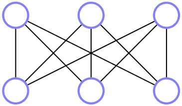

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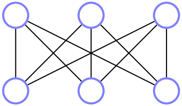

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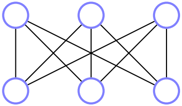

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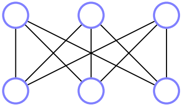

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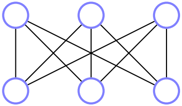

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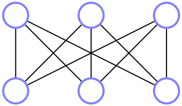

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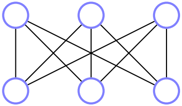
Proving non-planarity for $K_{3,3}$



Euler's formula $\implies$ $3f \leq 2e$ for any planar graph.

$K_{3,3}$? Edges? 9. Vertices. 6.

$9 \leq 3(6) - 6$? Sure!

Proof doesn't work. Let's fix this.

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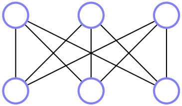

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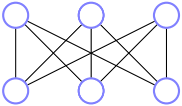

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Oh my goodness..what have we done!

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  Basics.

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 Non-planarity of K_5 and $K_{3,3}$.

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Yay!