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Induction and Recursion

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Induction and Recursion

Couple of more induction proofs.

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Couple of more induction proofs.

Stable Marriage.

Some quibbles.

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In some sense, the natural numbers.

Strong Induction and Recursion.

Thm: For every natural number $n \geq 12$, $n = 4x + 5y$.

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Slight differences: showed for all $n \geq 16$ that $\wedge_{i=4}^{n-1} P(i) \implies P(n)$.

Strengthening: need to...

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

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$$\sum_{i=1}^{k+1} \frac{1}{i^2}$$
$$= \sum_{i=1}^{k} \frac{1}{i^2} + \frac{1}{(k+1)^2}.$$
$$\leq 2 + \frac{1}{(k+1)^2}$$

Uh oh?

Hmmm... It better be that any sum is *strictly less than* 2.

How much less? At least by $\frac{1}{(k+1)^2}$ for S_k.

"$S_k \leq 2 - \frac{1}{(k+1)^2}$" $\Longrightarrow$ "$S_{k+1} \leq 2$"
Induction step works! No! Not the same statement!!!!
Need to prove "$S_{k+1} \leq 2 - \frac{1}{(k+2)^2}$".

Darn!!!

Strenthening: how?

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - f(n)$. ($S_n = \sum_{i=1}^{n} \frac{1}{i^2}$.)

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Proof:

Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

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Prove: $P(k+1)$

Strenthening: how?

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Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

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Proof:

Ind hyp: $P(k)$ — "$S_k \leq 2 - f(k)$"

Prove: $P(k+1)$ – "$S_{k+1} \leq 2 - f(k+1)$"

$$S(k+1) = S_k + \frac{1}{(k+1)^2}$$

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Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n}$.

Stable Marriage Problem

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- Small town with n boys and n girls.

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- Small town with n boys and n girls.
- Each girl has a ranked preference list of boys.

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- Small town with n boys and n girls.
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- Each boy has a ranked preference list of girls.

Stable Marriage Problem

- Small town with n boys and n girls.
- Each girl has a ranked preference list of boys.
- Each boy has a ranked preference list of girls.

How should they be matched?

Count the ways..

- Maximize total satisfaction.

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- Maximize number of first choices.

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- Maximize worse off.

Count the ways..

- Maximize total satisfaction.
- Maximize number of first choices.
- Maximize worse off.
- Minimize difference between preference ranks.

The best laid plans..

Consider the couples..

- Jennifer and Brad
- Angelina and Billy-Bob

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Brad prefers Angelina to Jennifer.

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Consider the couples..

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Brad prefers Angelina to Jennifer.

Angelina prefers Brad to BillyBob.

The best laid plans..

Consider the couples..

- Jennifer and Brad
- Angelina and Billy-Bob

Brad prefers Angelina to Jennifer.

Angelina prefers Brad to BillyBob.

Uh..oh.

So..

Produce a pairing where there is no running off!

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Definition: A **pairing** is disjoint set of n boy-girl pairs.

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Example: A pairing $S = \{(Brad, Jen); (BillyBob, Angelina)\}$.

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Definition: A **rogue couple** b, g^* for a pairing S:
b and g^* prefer each other to their partners in S

So..

Produce a pairing where there is no running off!

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Example: A pairing $S = \{(Brad, Jen); (BillyBob, Angelina)\}$.

Definition: A **rogue couple** b, g^* for a pairing S:
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Example: Brad and Angelina are a rogue couple in S.

A stable pairing??

Given a set of preferences.

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Is there a stable pairing?
How does one find it?

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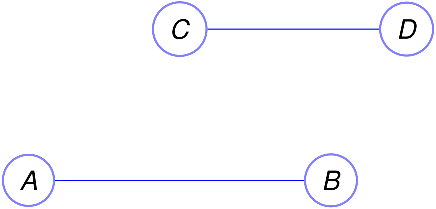
Consider a single gender version: stable roommates.

A	B	C	D
B	C	A	D
C	A	B	D
D	A	B	C



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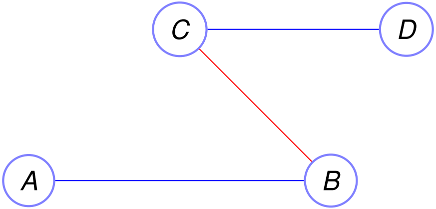

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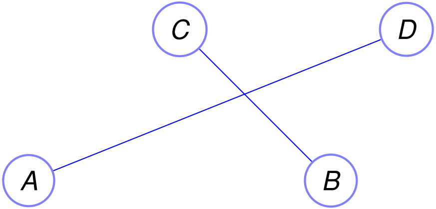

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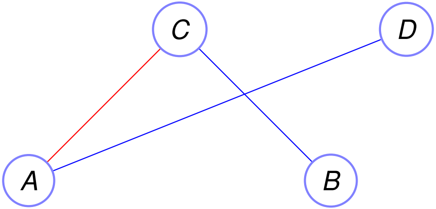

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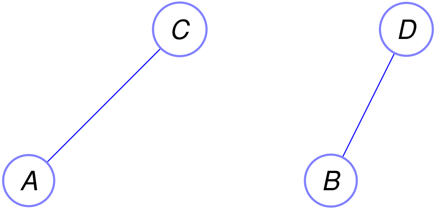

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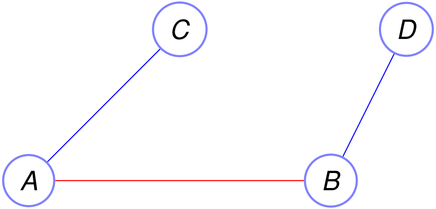

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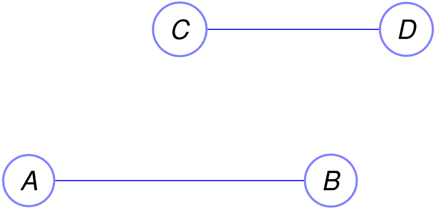

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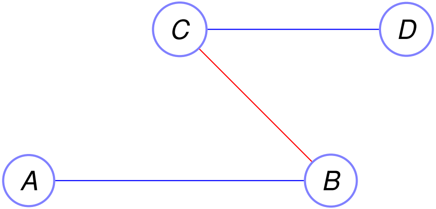
C	A	B	D
D	A	B	C



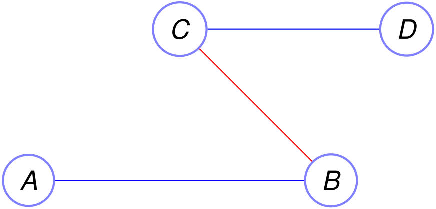
A stable pairing??

Given a set of preferences.

Is there a stable pairing?
How does one find it?

Consider a single gender version: stable roommates.

A	B	C	D
B	C	A	D
C	A	B	D
D	A	B	C



The Traditional Marriage Algorithm.

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Each Day:

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Each Day:

1. Each boy **proposes** to his favorite girl on his list.

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1. Each boy **proposes** to his favorite girl on his list.
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   (whom she puts on a string.)

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1. Each boy **proposes** to his favorite girl on his list.
2. Each girl rejects all but her favorite proposer (whom she puts on a string.)
3. Rejected boy crosses rejecting girl off his list.

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Stop when each girl gets exactly one proposal.

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Each Day:

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2. Each girl rejects all but her favorite proposer (whom she puts on a string.)
3. Rejected boy crosses rejecting girl off his list.

Stop when each girl gets exactly one proposal.
Does this terminate?

The Traditional Marriage Algorithm.

Each Day:

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Stop when each girl gets exactly one proposal.
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...produce a pairing?

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...produce a pairing?

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Do boys or girls do “better”?

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3. Rejected boy crosses rejecting girl off his list.

Stop when each girl gets exactly one proposal.
Does this terminate?

...produce a pairing?

....a stable pairing?

Do boys or girls do “better”?

Example.

Boys			
A	1	2	3
B	1	2	3
C	2	1	3

Girls			
1	C	A	B
2	A	B	C
3	A	C	B

Example.

Boys			
A	1	2	3
B	1	2	3
C	2	1	3

Girls			
1	C	A	B
2	A	B	C
3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1					
2					
3					

Example.

Boys			
A	1	2	3
B	1	2	3
C	2	1	3

Girls			
1	C	A	B
2	A	B	C
3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, B				
2	C				
3					

Example.

Boys			
A	1	2	3
B	~~1~~	2	3
C	2	1	3

Girls			
1	C	A	B
2	A	B	C
3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~				
2	C				
3					

Example.

	Boys		
A	1	2	3
B	~~1~~	2	3
C	2	1	3

	Girls		
1	C	A	B
2	A	B	C
3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~	A			
2	C	B, C			
3					

Example.

Boys			
A	1	2	3
B	~~1~~	2	3
C	~~2~~	1	3

Girls			
1	C	A	B
2	A	B	C
3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~	A			
2	C	B, ~~C~~			
3					

Example.

Boys			
A	1	2	3
B	~~1~~	2	3
C	~~2~~	1	3

Girls			
1	C	A	B
2	A	B	C
3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~	A	A , C		
2	C	B, ~~C~~	B		
3					

Example.

Boys			
A	~~1~~	2	3
B	~~1~~	2	3
C	~~2~~	1	3

Girls			
1	C	A	B
2	A	B	C
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	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~	A	~~A~~ , C		
2	C	B, ~~C~~	B		
3					

Example.

Boys			
A	~~1~~	2	3
B	~~1~~	2	3
C	~~2~~	1	3

Girls			
1	C	A	B
2	A	B	C
3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~	A	~~A~~ , C	C	
2	C	B, ~~C~~	B	A,B	
3					

Example.

Boys			
A	~~1~~	2	3
B	~~1~~	~~2~~	3
C	~~2~~	1	3

Girls			
1	C	A	B
2	A	B	C
3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~	A	~~A~~ , C	C	
2	C	B, ~~C~~	B	A, ~~B~~	
3					

Example.

Boys

A	~~1~~	2	3
B	~~1~~	~~2~~	3
C	~~2~~	1	3

Girls

1	C	A	B
2	A	B	C
3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~	A	~~A~~, C	C	C
2	C	B, ~~C~~	B	A, ~~B~~	A
3					B

Example.

Boys			
A	~~1~~	2	3
B	~~1~~	~~2~~	3
C	~~2~~	1	3

Girls			
1	C	A	B
2	A	B	C
3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~	A	~~A~~ , C	C	C
2	C	B, ~~C~~	B	A, ~~B~~	A
3					B

Termination.

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Every non-terminated day a boy **crossed** an item off the list.

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Total size of lists?

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Total size of lists? n boys, n length list.

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Total size of lists? n boys, n length list. n^2

Termination.

Every non-terminated day a boy **crossed** an item off the list.

Total size of lists? n boys, n length list. n^2

Terminates in at most $n^2 + 1$ steps!

It gets better every day for girls..

It gets better every day for girls..

Improvement Lemma: It just gets better for girls.

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Improvement Lemma: It just gets better for girls.

If on day t a girl g has a boy b on a string,

It gets better every day for girls..

Improvement Lemma: It just gets better for girls.

If on day t a girl g has a boy b on a string,
any boy, b', on g's string for any day $t' > t$

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If on day t a girl g has a boy b on a string,
any boy, b', on g's string for any day $t' > t$
is at least as good as b.

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Improvement Lemma: It just gets better for girls.

If on day t a girl g has a boy b on a string,
any boy, b', on g's string for any day $t' > t$
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Let's apply lemma.

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Let's apply lemma.

Girl "Alice" has boy "Bob" on string on day 5.

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Let's apply lemma.

Girl "Alice" has boy "Bob" on string on day 5.

She has boy "Jim" on string on day 7.

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Let's apply lemma.

Girl "Alice" has boy "Bob" on string on day 5.

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Does Alice prefer "Jim" or "Bob"?

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Girl "Alice" has boy "Bob" on string on day 5.

She has boy "Jim" on string on day 7.

Does Alice prefer "Jim" or "Bob"?

g - 'Alice', b - 'Bob', b' - 'Jim', $t = 5$, $t' = 7$.

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Girl "Alice" has boy "Bob" on string on day 5.

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Does Alice prefer "Jim" or "Bob"?

g - 'Alice', b - 'Bob', b' - 'Jim', $t = 5$, $t' = 7$.

Improvement Lemma says she prefers 'Jim'.

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Does Alice prefer "Jim" or "Bob"?

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On day 10, could Alice still have "Jim" on her string?

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On day 10, could Alice still have "Jim" on her string? Yes.

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On day 10, could Alice still have "Jim" on her string? Yes.

She likes her day 10 boy at least as much as her day 7 boy.
Here, $b = b'$.

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Why is lemma true?

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Why is lemma true?

Proof Idea:

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Let's apply lemma.

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Does Alice prefer “Jim” or “Bob”?

g - 'Alice', b - 'Bob', b' - 'Jim', $t = 5$, $t' = 7$.

Improvement Lemma says she prefers 'Jim'.

On day 10, could Alice still have “Jim” on her string? Yes.

She likes her day 10 boy at least as much as her day 7 boy.
Here, $b = b'$.

Why is lemma true?

Proof Idea: Because she can always keep the previous boy on the string.

Improvement Lemma

Improvement Lemma: It just gets better for girls.

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If on day *t* a girl *g* has a boy *b* on a string, any boy, b', on *g*'s string for any day $t' > t$ is at least as good as *b*.

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If on day t a girl g has a boy b on a string, any boy, b', on g's string for any day $t' > t$ is at least as good as b.

Proof:

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If on day t a girl g has a boy b on a string, any boy, b', on g's string for any day $t' > t$ is at least as good as b.

Proof:
$P(k)$- - "boy on g's string is at least as good as b on day $t+k$"

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Proof:
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$P(0)$– true. Girl has b on string.

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Assume $P(k)$. Let b' be boy **on string** on day $t+k$.

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$P(0)$– true. Girl has b on string.

Assume $P(k)$. Let b' be boy **on string** on day $t+k$.

On day $t+k+1$, boy b' comes back.

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Girl can choose b',

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Assume $P(k)$. Let b' be boy **on string** on day $t+k$.

On day $t+k+1$, boy b' comes back.
Girl can choose b', or do better with another boy, b''

Improvement Lemma

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That is,

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$P(0)$– true. Girl has b on string.

Assume $P(k)$. Let b' be boy **on string** on day $t+k$.

On day $t+k+1$, boy b' comes back.
Girl can choose b', or do better with another boy, b''

That is, $b' \leq b$ by induction hypothesis.

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$P(0)$– true. Girl has b on string.

Assume $P(k)$. Let b' be boy **on string** on day $t+k$.

On day $t+k+1$, boy b' comes back.
Girl can choose b', or do better with another boy, b''

That is, $b' \leq b$ by induction hypothesis.
And b'' is better than b' by algorithm.

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Assume $P(k)$. Let b' be boy **on string** on day $t+k$.

On day $t+k+1$, boy b' comes back.
Girl can choose b', or do better with another boy, b''

That is, $b' \leq b$ by induction hypothesis.
And b'' is better than b' by algorithm.
$\implies$ Girl does at least as well as with b.

Improvement Lemma

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$P(k) \implies P(k+1)$.

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$P(k) \implies P(k+1)$. And by principle of induction.

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$P(k) \implies P(k+1)$. And by principle of induction. $\square$

Pairing when done.

Lemma: Every boy is matched at end.

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$\implies$ each girl has a boy on a string.

Pairing when done.

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Every girl has been proposed to by b,
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$\implies$ each girl has a boy on a string.

and each boy is on at most one string.

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$\implies$ each girl has a boy on a string.

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n girls and n boys.

Pairing when done.

Lemma: Every boy is matched at end.

Proof:
If not, a boy b must have been rejected n times.

Every girl has been proposed to by b,
and Improvement lemma

$\Longrightarrow$ each girl has a boy on a string.

and each boy is on at most one string.

n girls and n boys. Same number of each.

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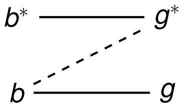

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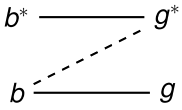

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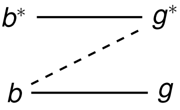

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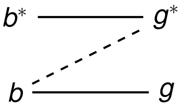

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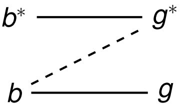

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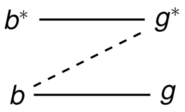

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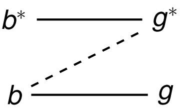

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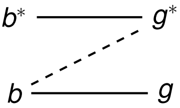

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A:	1,2	1:	B,A
B:	2,1	2:	A,B

Pairing S: $(A,1),(B,2)$. Stable? Yes.

Pairing T: $(A,2),(B,1)$. Also Stable.

Which is optimal for A? S Which is optimal for B? S
Which is optimal for 1? T Which is optimal for 2?

Understanding Optimality: by example.

A:	1,2	1:	A,B
B:	1,2	2:	B,A

Consider pairing: $(A,1),(B,2)$.

Stable? Yes.

Optimal for B?
Notice: only one stable pairing.
So this is the best B can do in a stable pairing.
So optimal for B.

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For boys?

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Theorem: TMA produces a boy-optimal pairing.

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Proof:

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Used Well-Ordering principle...Induction.

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Structural statement: Boy optimality

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Notes: Not really induction.
Structural statement: Boy optimality $\implies$ Girl pessimality.

Quick Questions.

How does one make it better for girls?

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SMA - stable marriage algorithm. One side proposes.

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SMA - stable marriage algorithm. One side proposes.
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SMA - stable marriage algorithm. One side proposes.
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Girls could propose.

Quick Questions.

How does one make it better for girls?

SMA - stable marriage algorithm. One side proposes.
TMA - boys propose.
Girls could propose. $\implies$ optimal for girls.

Residency Matching..

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The method was used to match residents to hospitals.

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Hospital optimal....

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Another variation: couples.

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Don’t go!

Summary.

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▸ Link