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Drink $\implies$ "≥ 18"

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Which cards do you turn over?

Drink $\implies$ "≥ 18"

"< 18" $\implies$ Don't Drink.

CS70: Lecture 2. Outline.

Today: Proofs!!!

1. By Example.
2. Direct. (Prove $P \implies Q$.)
3. by Contraposition (Prove $P \implies Q$)
4. by Contradiction (Prove P.)
5. by Cases

If time: discuss induction.

Quick Background and Notation.

Integers closed under addition.

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A natural number $p > 1$, is **prime** if it is divisible only by 1 and itself.

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Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

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Assume P.
...
Therefore Q.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

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$n = 121$ Alt Sum: $1 - 2 + 1 = 0$.

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Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

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Add $99a + 11b$ to both sides.

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Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Left hand side is n, $k + 9a + b$ is integer. $\implies 11|n$. $\square$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

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Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

$n = 2k + 1$

$n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1.$

$n^2 = 2l + 1$ where l is a natural number..

... and n^2 is odd!

$\neg Q \implies \neg P$

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

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- Proof assumed no primes *in between* p_k and q.

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.

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Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

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Reduced form $\frac{a}{b}$: a and b can't both be even!

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Case 2: a even, b odd: even - even +odd = even. Not possible.

Proof by cases.

Theorem: $x^5 - x + 1 = 0$ has no solution in the rationals.
Proof: First a lemma...

Lemma: If x is a solution to $x^5 - x + 1 = 0$ and $x = a/b$ for $a, b \in Z$, *then both a and b are even.*

Reduced form $\frac{a}{b}$: a and b can't both be even! + Lemma
$\implies$ no rational solution. $\square$

Proof of lemma: Assume a solution of the form a/b.

$$\left(\frac{a}{b}\right)^5 - \frac{a}{b} + 1 = 0$$

Multiply by b^5,

$$a^5 - ab^4 + b^5 = 0$$

Case 1: a odd, b odd: odd - odd +odd = even. Not possible.
Case 2: a even, b odd: even - even +odd = even. Not possible.
Case 3: a odd, b even: odd - even +even = even.

Proof by cases.

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The fourth case is the only one possible,

Proof by cases.

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Case 4: a even, b even: even - even +even = even. Possible.

The fourth case is the only one possible, so the lemma follows. $\square$

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

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Let $x = y = \sqrt{2}$.

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Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational.

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

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Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

Proof by cases.

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Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

- New values: $x = \sqrt{2}^{\sqrt{2}}$, $y = \sqrt{2}$.

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-

$$x^y =$$

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Thus, we have irrational x and y with a rational x^y (i.e., 2).

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One of the cases is true so theorem holds.

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One of the cases is true so theorem holds. □

Question: Which case holds?

Proof by cases.

Theorem: There exist irrational x and y such that x^y is rational.

Let $x = y = \sqrt{2}$.

Case 1: $x^y = \sqrt{2}^{\sqrt{2}}$ is rational. Done!

Case 2: $\sqrt{2}^{\sqrt{2}}$ is irrational.

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$$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}*\sqrt{2}} = \sqrt{2}^{2} = 2.$$

Thus, we have irrational x and y with a rational x^y (i.e., 2).

One of the cases is true so theorem holds. $\square$

Question: Which case holds? Don't know!!!

Be careful.

Theorem: $3 = 4$

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Start with $12 = 12$.

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Divide one side by 3 and the other by 4 to get
$$4 = 3.$$

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By commutativity

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By commutativity theorem holds. $\square$

Be careful.

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Proof: Assume $3 = 4$.

Start with $12 = 12$.

Divide one side by 3 and the other by 4 to get
$4 = 3$.

By commutativity theorem holds. $\square$

Don't assume what you want to prove!

Be really careful!

Theorem: $1 = 2$
Proof:

Be really careful!

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Proof: For $x = y$, we have

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Theorem: $1 = 2$
Proof: For $x = y$, we have

$$
\begin{aligned}
(x^2 - xy) &= x^2 - y^2 \\
x(x - y) &= (x + y)(x - y)
\end{aligned}
$$

Be really careful!

Theorem: $1 = 2$

Proof: For $x = y$, we have

$$\begin{aligned}(x^2 - xy) &= x^2 - y^2 \\ x(x-y) &= (x+y)(x-y) \\ x &= (x+y)\end{aligned}$$

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$$\begin{aligned}
(x^2 - xy) &= x^2 - y^2 \\
x(x - y) &= (x + y)(x - y) \\
x &= (x + y) \\
x &= 2x
\end{aligned}$$

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Dividing by zero is no good.

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Also: Multiplying inequalities by a negative.

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Also: Multiplying inequalities by a negative.

$P \implies Q$ does not mean $Q \implies P$.

Summary: Note 2.

Direct Proof:

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Direct Proof:
To Prove: $P \implies Q$.

Summary: Note 2.

Direct Proof:

To Prove: $P \implies Q$. Assume P.

Summary: Note 2.

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Summary: Note 2.

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Summary: Note 2.

Direct Proof:
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By Contraposition:
To Prove: $P \implies Q$ Assume $\neg Q$.

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To Prove: P Assume $\neg P$. Prove False .

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By Cases: informal.

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To Prove: P Assume $\neg P$. Prove False .

By Cases: informal.
Universal: show that statement holds in all cases.

Summary: Note 2.

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To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

By Contradiction:
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By Cases: informal.
Universal: show that statement holds in all cases.
Existence: used cases where one is true.

Summary: Note 2.

Direct Proof:
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Either $\sqrt{2}$ and $\sqrt{2}$ worked.

Summary: Note 2.

Direct Proof:
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Careful when proving!

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 Universal: show that statement holds in all cases.
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Careful when proving!
  Don't assume the theorem.

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By Cases: informal.
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    Either $\sqrt{2}$ and $\sqrt{2}$ worked.
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Careful when proving!
    Don't assume the theorem. Divide by zero.

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To Prove: P Assume $\neg P$. Prove False .

By Cases: informal.
Universal: show that statement holds in all cases.
Existence: used cases where one is true.
Either $\sqrt{2}$ and $\sqrt{2}$ worked.
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Careful when proving!
Don't assume the theorem. Divide by zero.Watch converse.

Summary: Note 2.

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To Prove: $P \implies Q$. Assume P. Prove Q.

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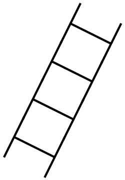
Don't assume the theorem. Divide by zero.Watch converse. ...

CS70: Note 3. Induction!

1. The natural numbers.
2. 5 year old Gauss.
3. ..and Induction.
4. Simple Proof.

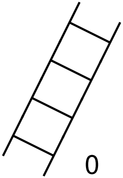
The natural numbers.

The natural numbers.



The natural numbers.

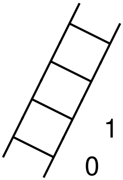
0,



0

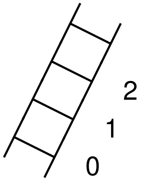
The natural numbers.

0, 1,



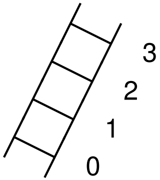
The natural numbers.

0, 1, 2,



The natural numbers.

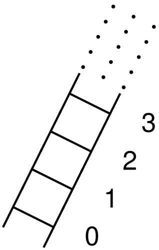
0, 1, 2, 3,



The natural numbers.

0, 1, 2, 3,

$\ldots,$



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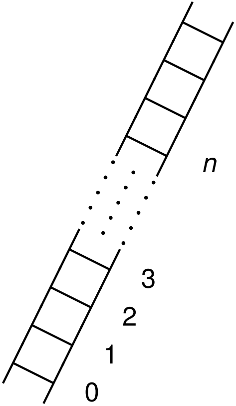

0, 1, 2, 3,

…, n,

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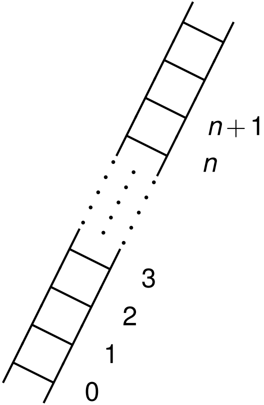

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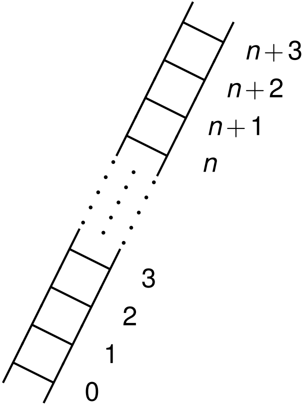

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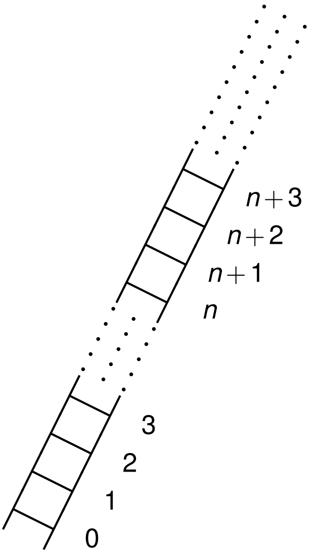

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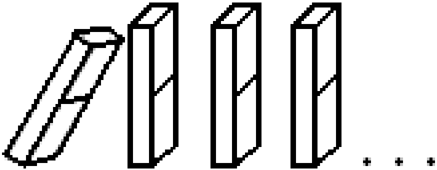
Notes visualization

Note's visualization: an infinite sequence of dominos.



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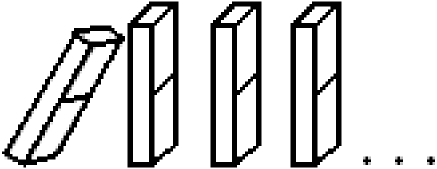

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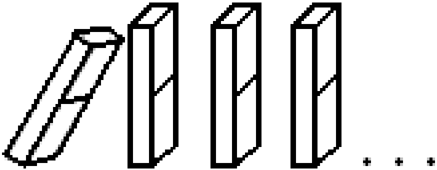

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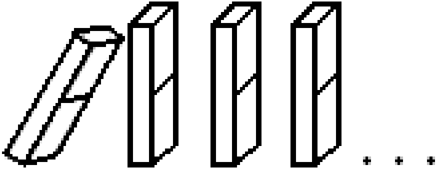

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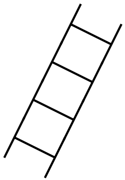

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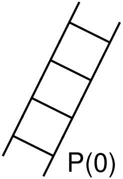

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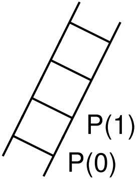

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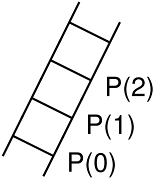

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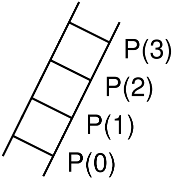

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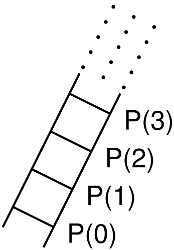

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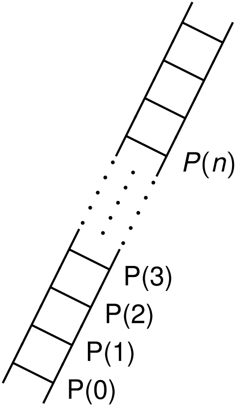

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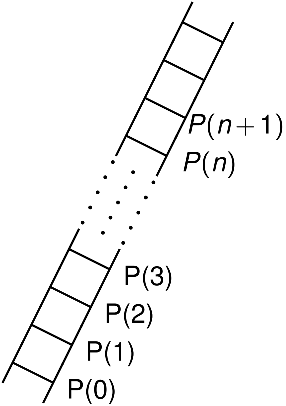

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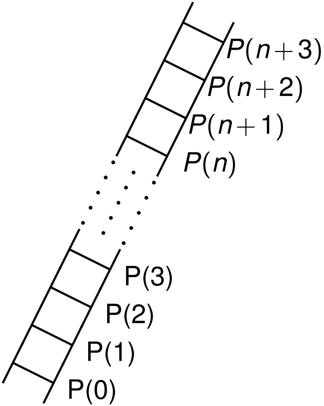

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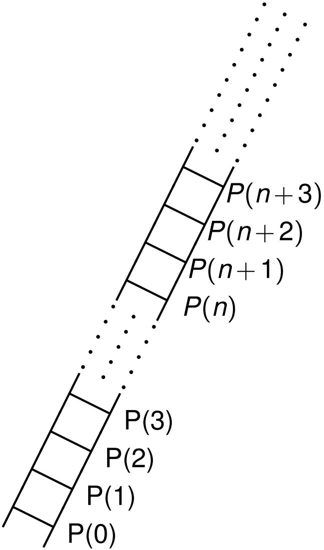

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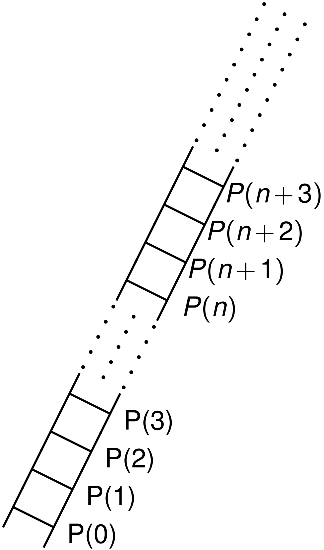

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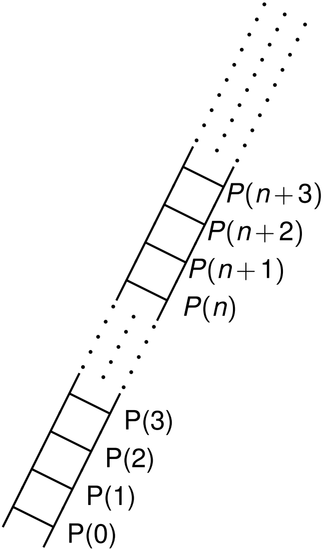

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Child Gauss: $(\forall \mathbf{n} \in \mathbf{N})(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$

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Predicate, $P(n)$, True for all natural numbers!

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Predicate, $P(n)$, True for all natural numbers! **Proof by Induction.**

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

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- For all natural numbers n, $1 + 2 \cdots n = \frac{n(n+1)}{2}$.

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- For all natural numbers n, $1 + 2 \cdots n = \frac{n(n+1)}{2}$.
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- The sum of the first n odd integers is a perfect square.

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- Prove $P(0)$. “Base Case”.

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Induction

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$P(n)$ true for all natural numbers n!!!

Induction

The canonical way of proving statements of the form

$$(\forall k \in N)(P(k))$$

- For all natural numbers n, $1+2\cdots n = \frac{n(n+1)}{2}$.
- For all $n \in N$, $n^3 - n$ is divisible by 3.
- The sum of the first n odd integers is a perfect square.

The basic form

- Prove $P(0)$. “Base Case”.
- $P(k) \implies P(k+1)$
  - Assume $P(k)$, “Induction Hypothesis”
  - Prove $P(k+1)$. “Induction Step.”

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Next Time.

More induction!

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See you on Tuesday!