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Disjoint – so add!

CS70: On to probability.

Modeling Uncertainty: Probability Space

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Modeling Uncertainty: Probability Space

1. Key Points
2. Random Experiments
3. Probability Space

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The Magic of Probability

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Uncertainty:

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Your cost: focused attention and practice on examples and problems.

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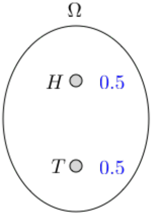
Physical Experiment



Probability Model

Random Experiment: Flip one Fair Coin

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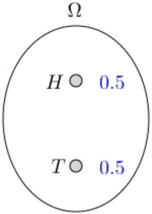

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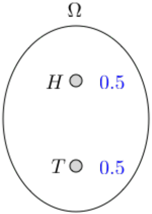

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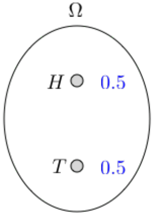

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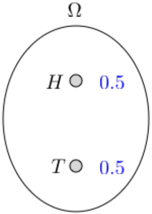

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  - A set Ω of outcomes: $\Omega = \{H, T\}$.

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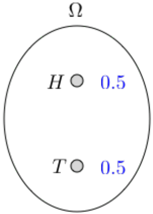
Flip a fair coin: model

Physical Experiment



Probability Model

- The physical experiment is complex. (Shape, density, initial momentum and position, ...)
- The Probability model is simple:
  - A set Ω of outcomes: $\Omega = \{H, T\}$.
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Random Experiment: Flip one Fair Coin

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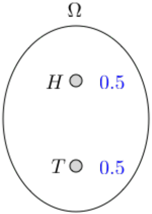

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Random Experiment: Flip one Unfair Coin

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Flip an unfair (biased, loaded) coin:

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Flip an unfair (biased, loaded) coin:

H: 45%

T: 55%

- Possible outcomes:

Random Experiment: Flip one Unfair Coin

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- Possible outcomes: Heads (H) and Tails (T)

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- Tautology? No: Statistical regularity!

Random Experiment: Flip one Unfair Coin

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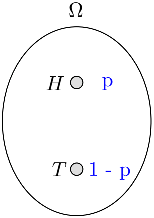

Probability Model

Flip Two Fair Coins

Flip Two Fair Coins

- Possible outcomes:

Flip Two Fair Coins

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Flip Two Fair Coins

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Flip Glued Coins

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Flips two coins glued together side by side:

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Glued coins

50%

50%

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- Possible outcomes:

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Flips two coins glued together side by side:

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Flip Glued Coins

Flips two coins glued together side by side:

Glued coins

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- Possible outcomes: $\{HT, TH\}$.
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- Note: Coins are glued so that they show different faces.

Flip two Attached Coins

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Flips two coins attached by a spring:

Flip two Attached Coins

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Attached coins

- Possible outcomes:

Flip two Attached Coins

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Flip two Attached Coins

Flips two coins attached by a spring:

- Possible outcomes: $\{HH, HT, TH, TT\}$.
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- Note: Coins are attached so that they tend to show the same face, unless the spring twists enough.

Flipping Two Coins

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Here is a way to summarize the four random experiments:

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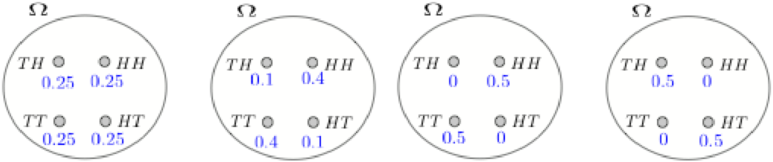

[1] [2] [3] [4]

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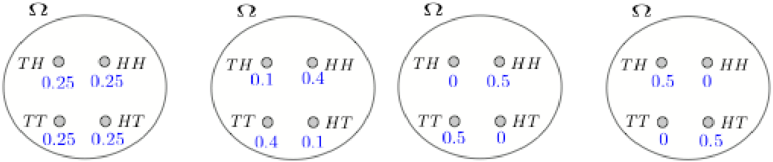

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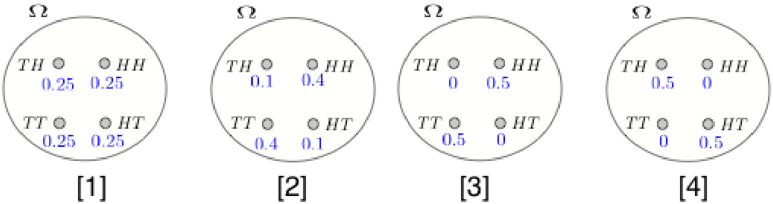

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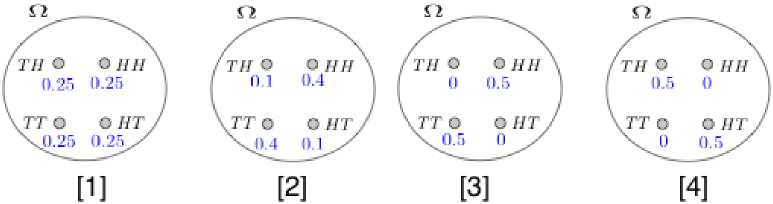

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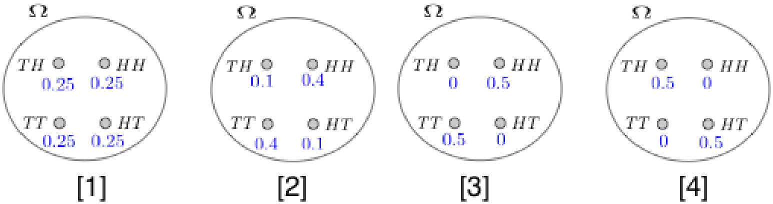

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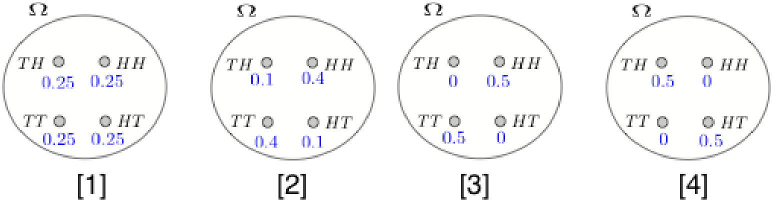

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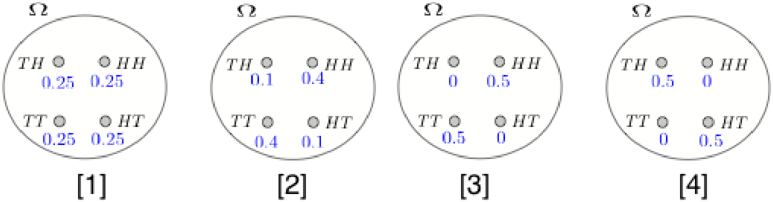

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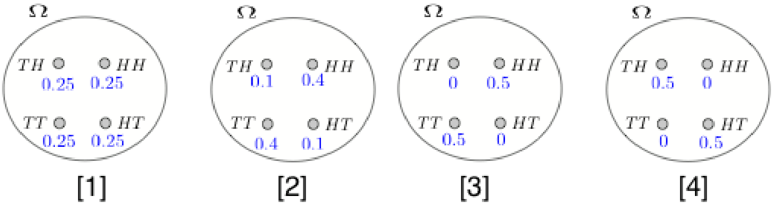

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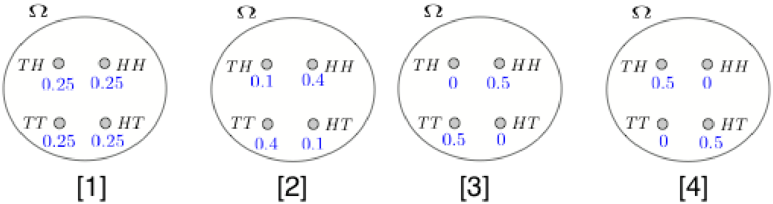

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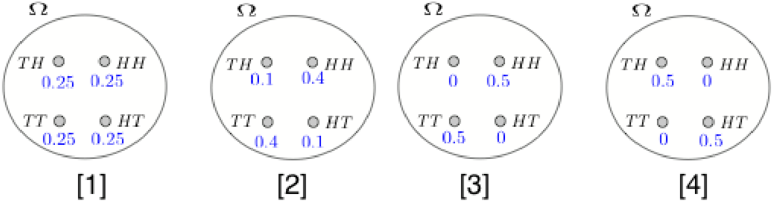

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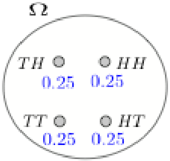

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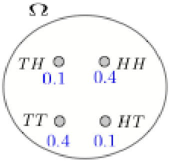

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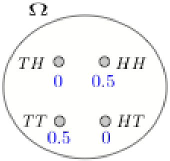

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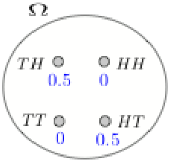

[4]

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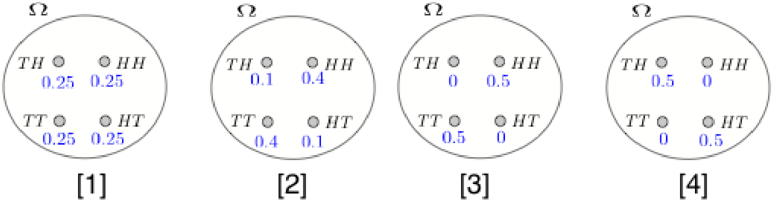

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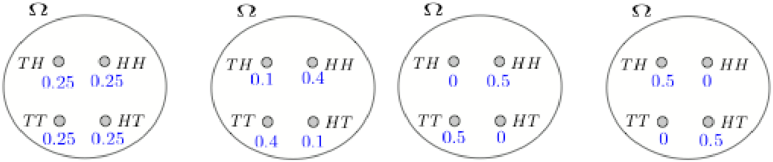

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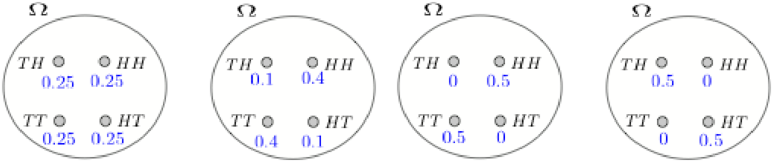

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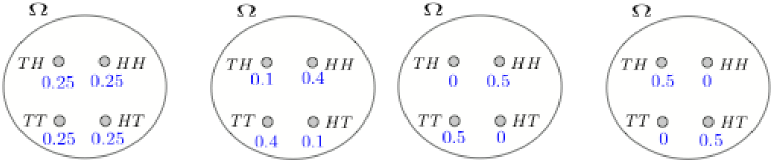

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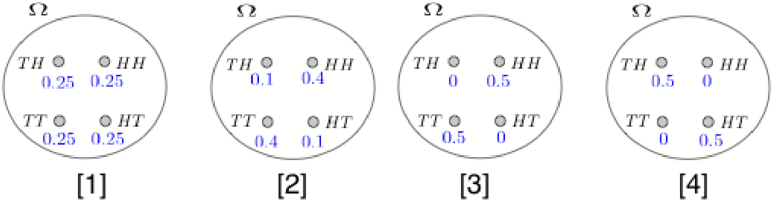

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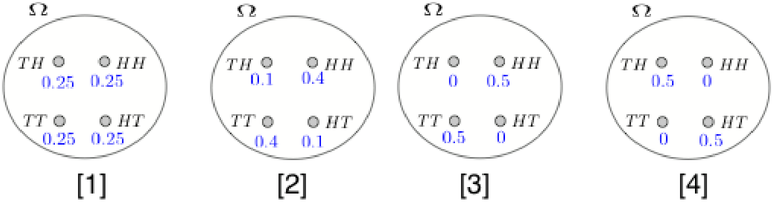

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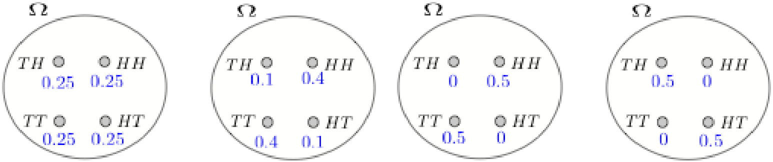

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- Ω and the probabilities specify the random experiment.

Flipping *n* times

Flip a fair coin *n* times (some $n \geq 1$):

Flipping n times

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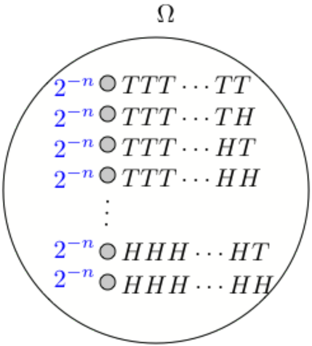

Roll two Dice

Roll a balanced 6-sided die twice:

Roll two Dice

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- Possible outcomes:

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Roll a balanced 6-sided die twice:

- Possible outcomes: $\{1,2,3,4,5,6\}^2 = \{(a,b) \mid 1 \leq a,b \leq 6\}$.

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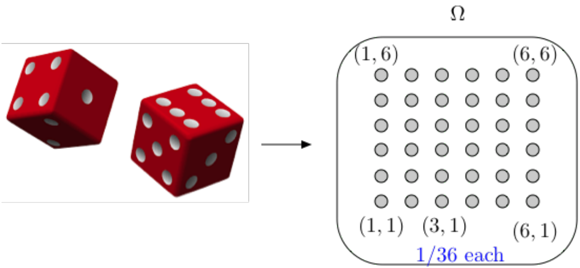

Physical Experiment   Probability Model

Probability Space.

1. A "random experiment":

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   (a) $\Omega = \{H, T\}$;
   (b) $\Omega = \{HH, HT, TH, TT\}$; $|\Omega| =$

Probability Space.

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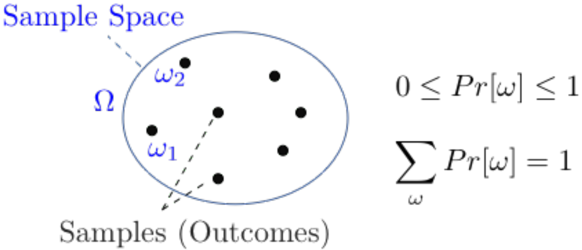

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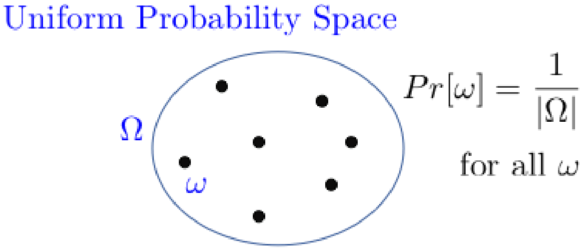

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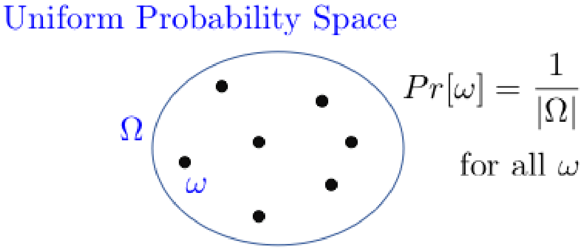

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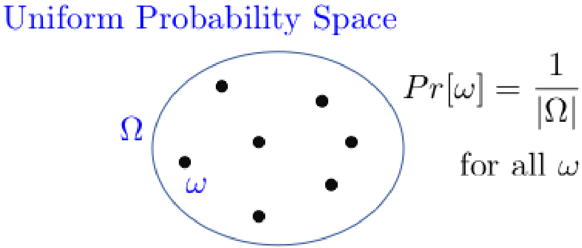

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Simplest physical model of a uniform probability space:

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Simplest physical model of a uniform probability space:

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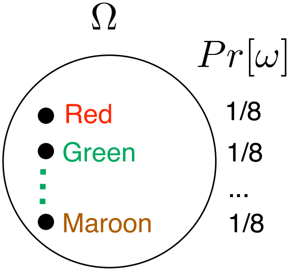

Probability model

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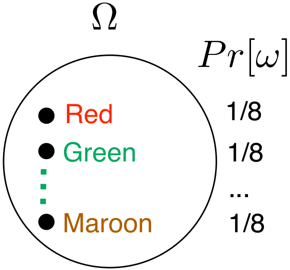

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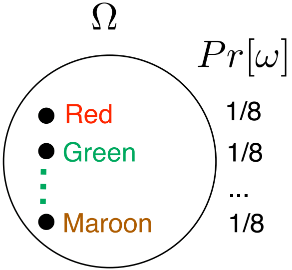

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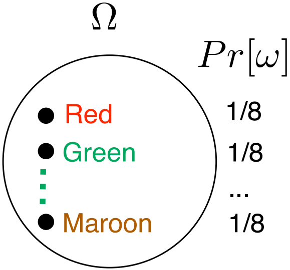

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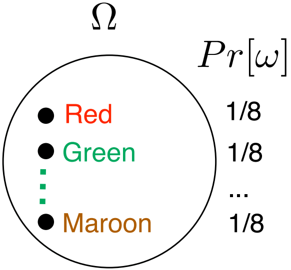

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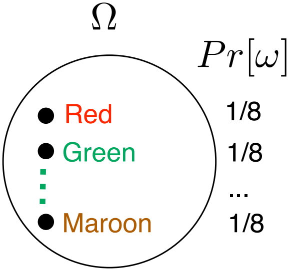

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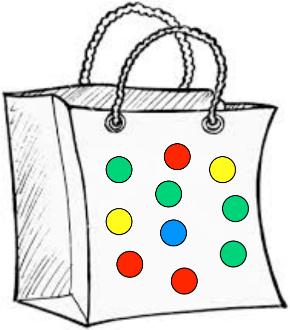

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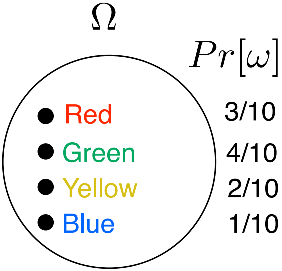

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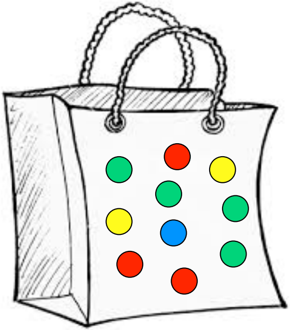

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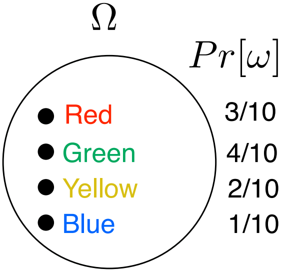

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$\Omega = \{$Red, Green, Yellow, Blue$\}$

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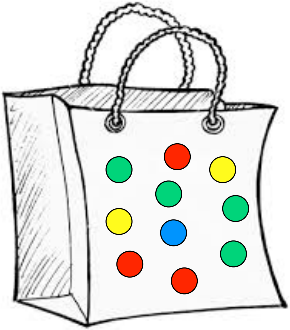

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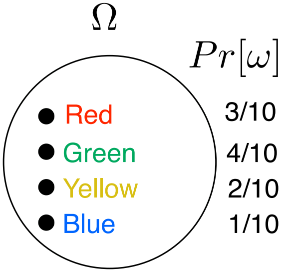

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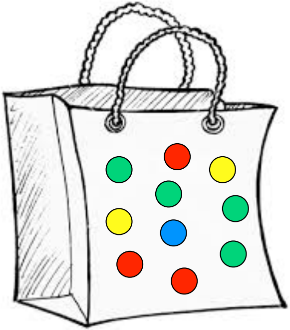

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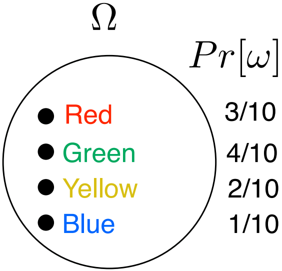

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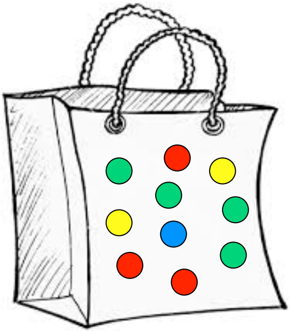

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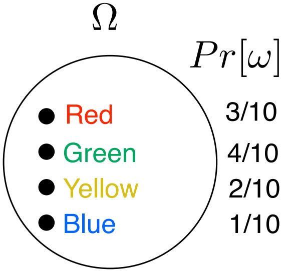

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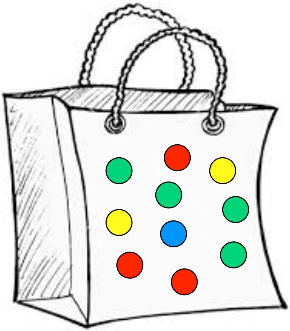

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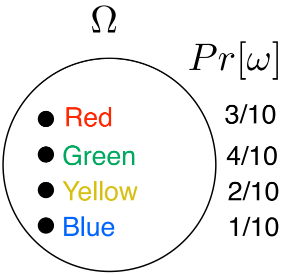

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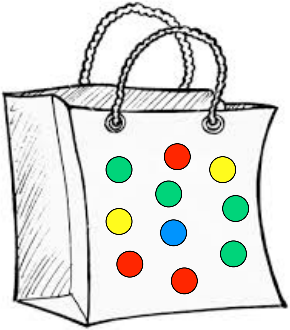

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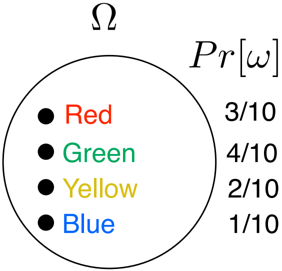

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Note: Probabilities are restricted to rational numbers: $\frac{N_k}{N}$.

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Physical model of a general non-uniform probability space:

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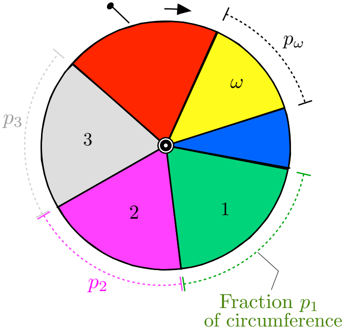

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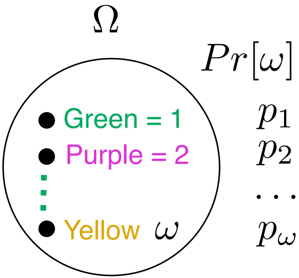

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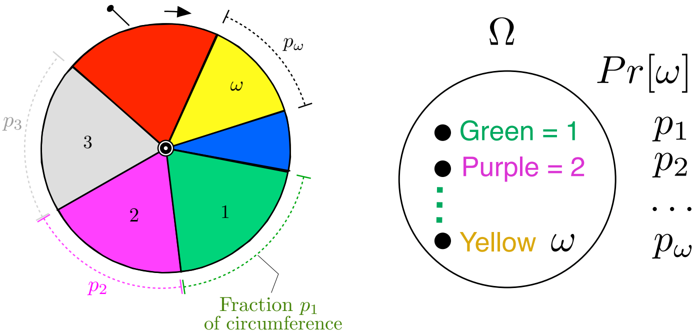

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The roulette wheel stops in sector ω with probability p_ω.

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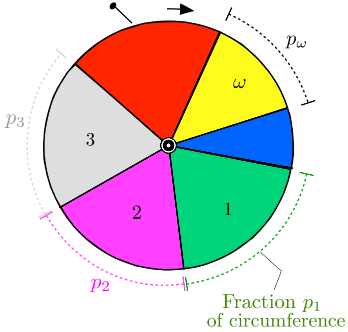

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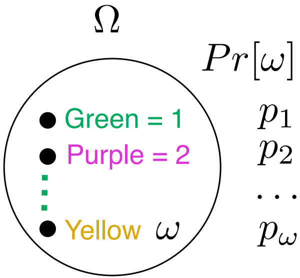

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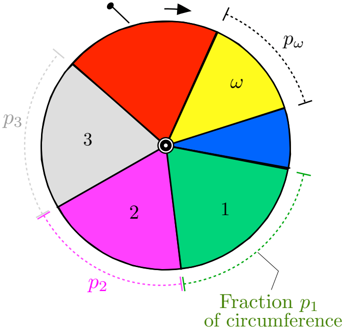

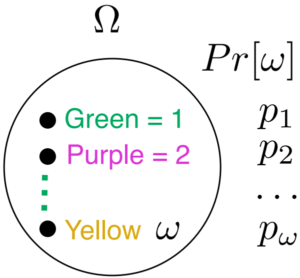

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Summary of Probability Basics

Modeling Uncertainty: Probability Space

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Onwards in Probability.

Events, Conditional Probability, Independence, Bayes' Rule

CS70: On to Events.

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Events, Conditional Probability, Independence, Bayes' Rule

Today: Events.

Probability Basics Review

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Set notation review

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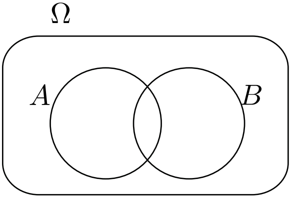
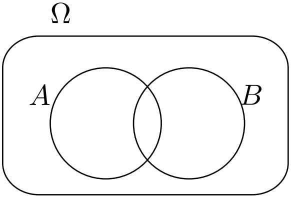


Figure : Two events

Set notation review



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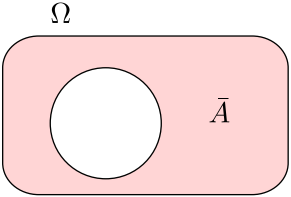
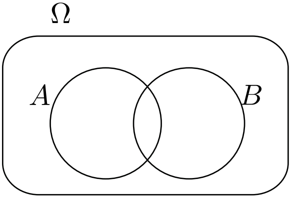


Figure : Complement (not)

Set notation review



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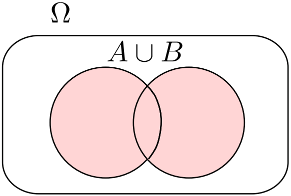
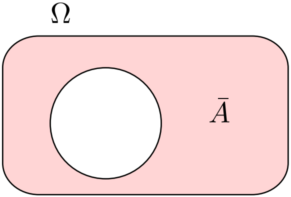


Figure : Union (or)



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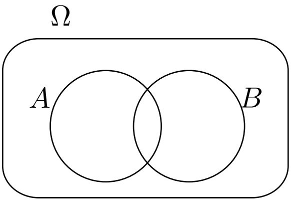

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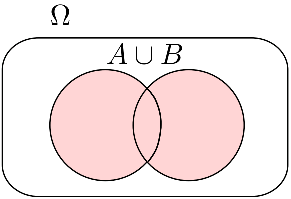

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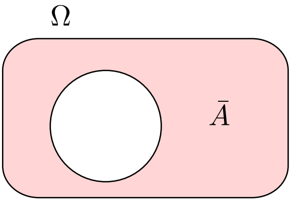

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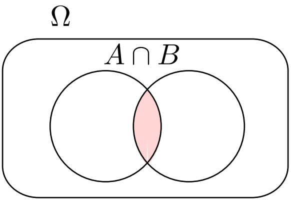

Figure : Intersection (and)

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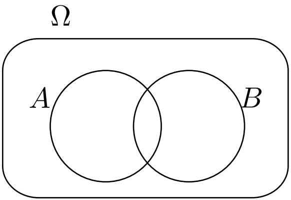

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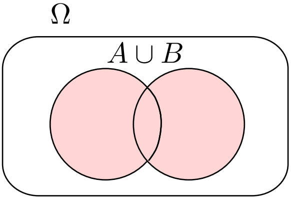

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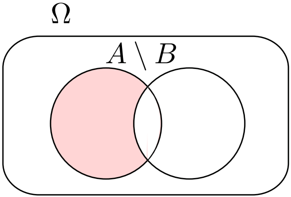
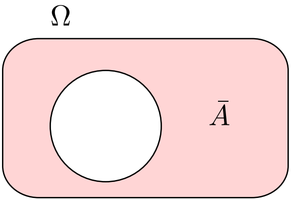


Figure : Difference (A, not B)



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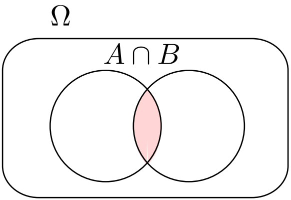

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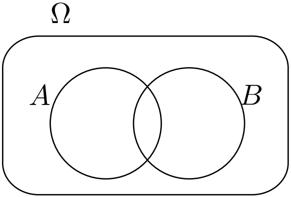

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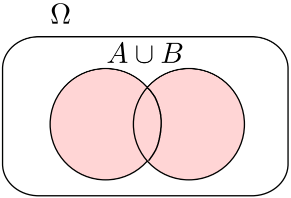

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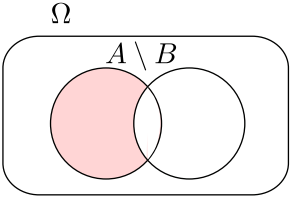

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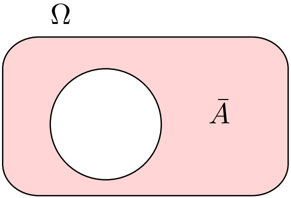

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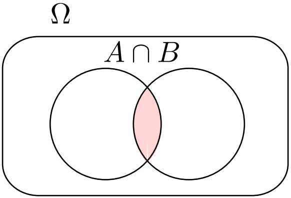

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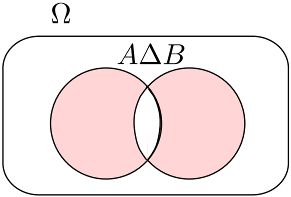

Figure : Symmetric difference (only one)

Probability of exactly one ‘heads’ in two coin flips?

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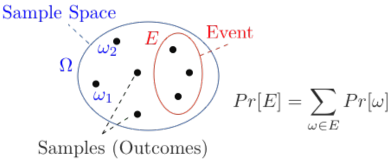

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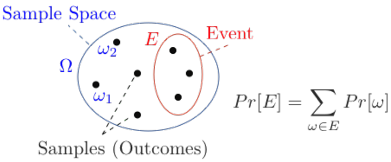

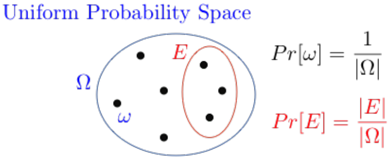

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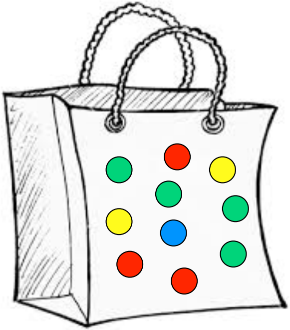
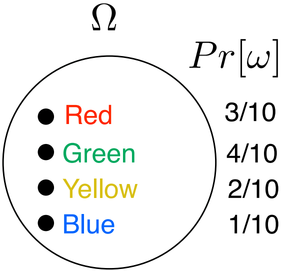


Physical experiment



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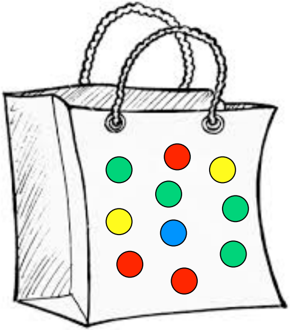

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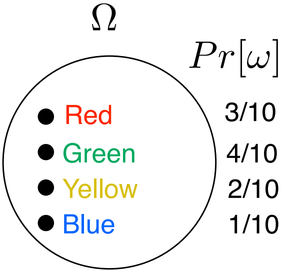

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$\Omega = \{$Red, Green, Yellow, Blue$\}$

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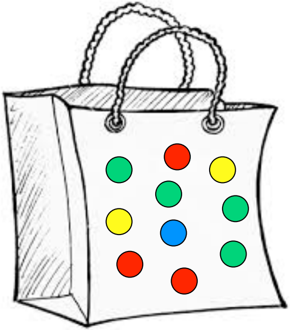

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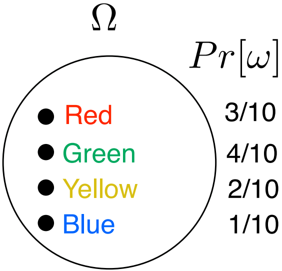

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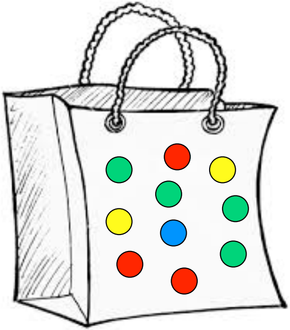

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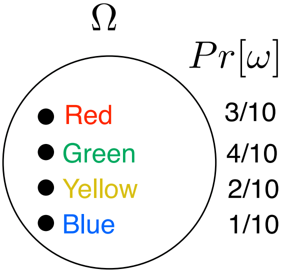

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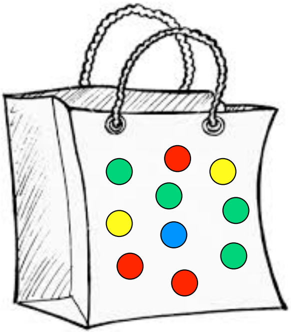

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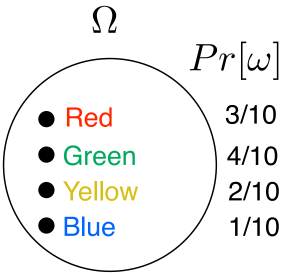

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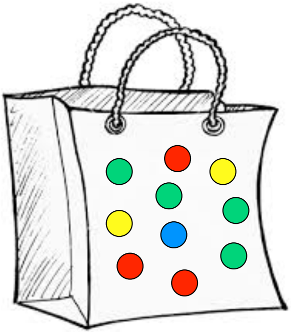

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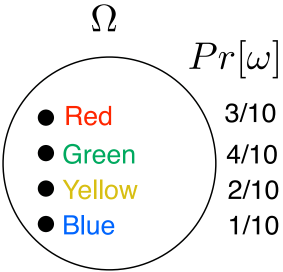

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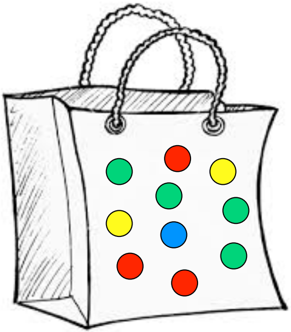

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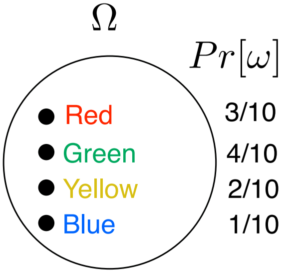

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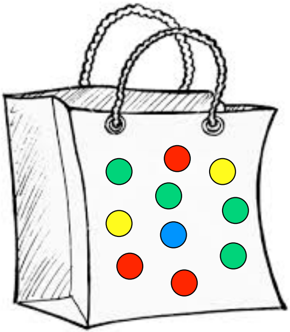

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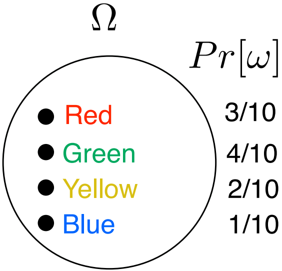

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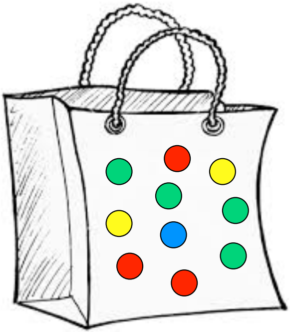

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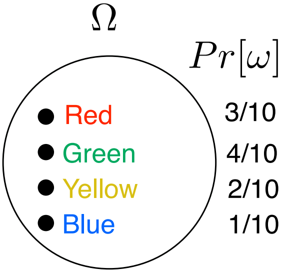

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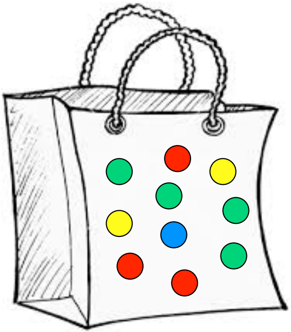

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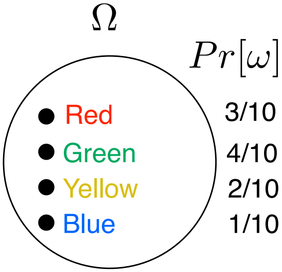

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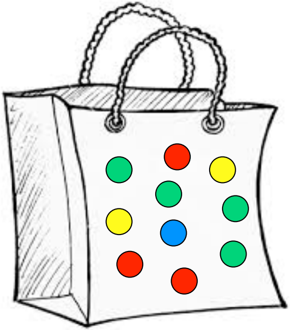

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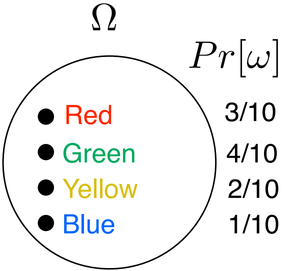

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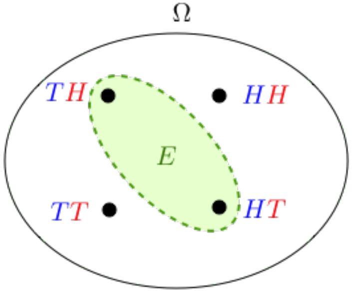

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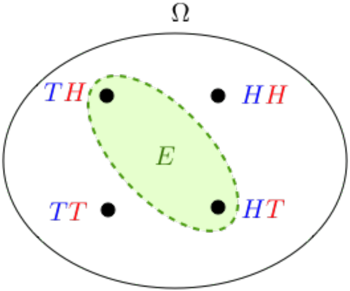

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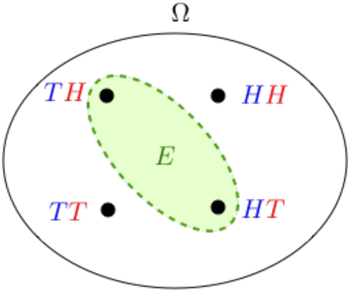

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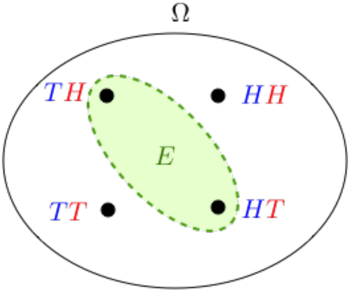

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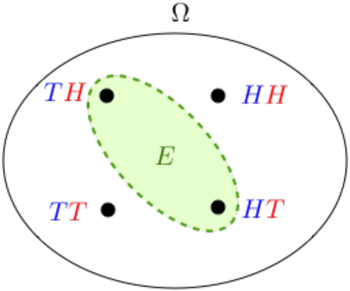

$$Pr[E] = \sum_{\omega \in E} Pr[\omega] = \frac{|E|}{|\Omega|} = \frac{2}{4} = \frac{1}{2}.$$

Roll a red and a blue die.

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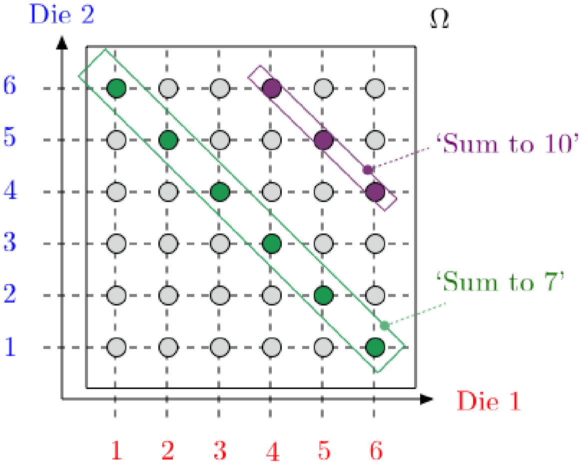

$Pr[\text{Sum to } 7] = \frac{6}{36}$ $Pr[\text{Sum to } 10] = \frac{3}{36}$

Example: 20 coin tosses.

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Sample space: $\Omega =$ set of 20 fair coin tosses.

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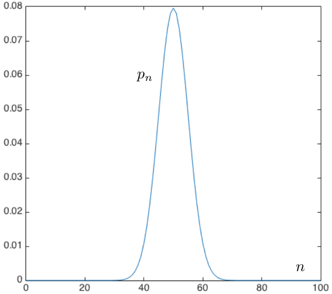

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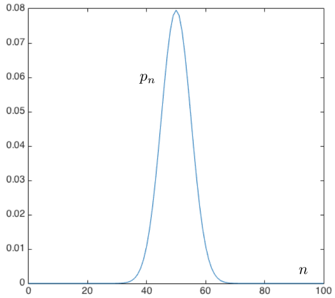

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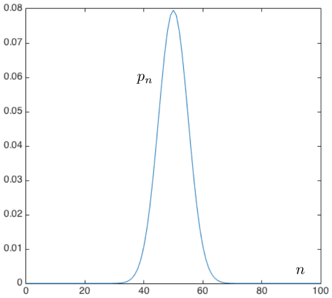

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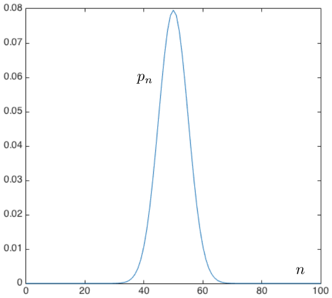

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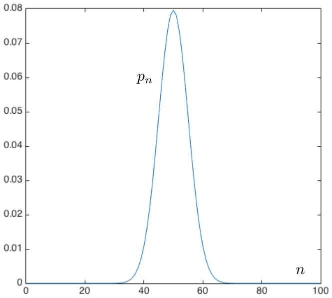

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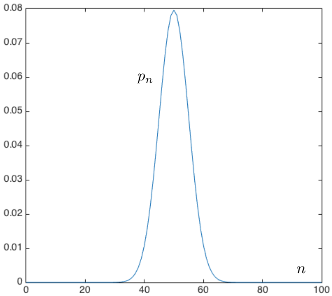

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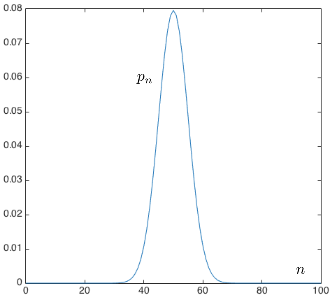

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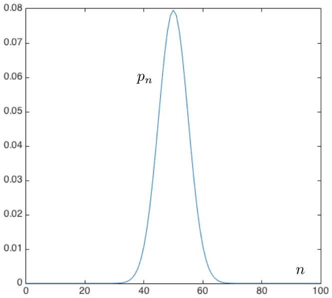

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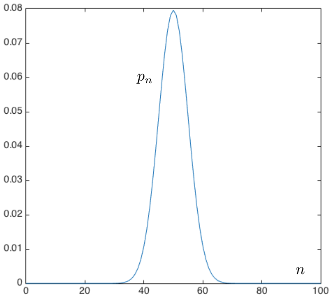

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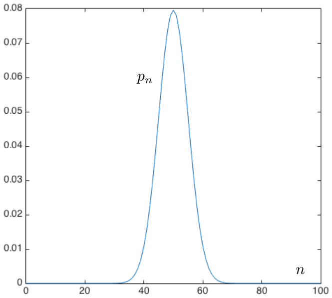

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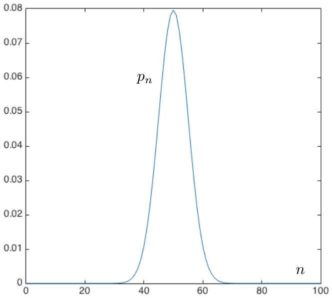

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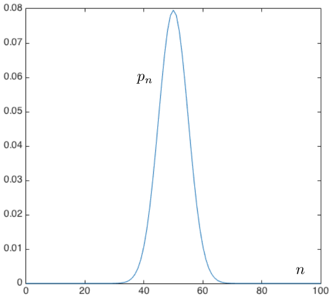

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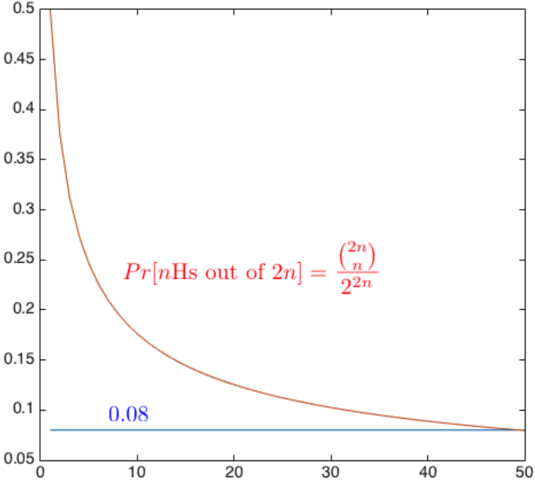
Stirling formula (for large n):

$$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n.$$

$$\binom{2n}{n} \approx \frac{\sqrt{4\pi n}(2n/e)^{2n}}{[\sqrt{2\pi n}(n/e)^n]^2} \approx \frac{4^n}{\sqrt{\pi n}}.$$

$$Pr[E] = \frac{|E|}{|\Omega|} = \frac{|E|}{2^{2n}} = \frac{1}{\sqrt{\pi n}} = \frac{1}{\sqrt{50\pi}} \approx .08.$$

Exactly 50 heads in 100 coin tosses.



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3. Uniform Probability Space: $Pr[\omega] = 1/|\Omega|$ for all $\omega \in \Omega$.
4. Event: "subset of outcomes." $A \subseteq \Omega$. $Pr[A] = \sum_{w \in A} Pr[\omega]$
5. Some calculations.