Today.

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  Shared (and sort of kept) secrets.

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Tolerated Loss: erasure codes.

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Error Correction!!!!

The mathematics.

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Assumption: a field, in particular, arithmetic $\mod p$.

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Arithmetic $\pmod p \implies$ work with $O(\log p)$ bit numbers.

Erasure Codes.

Satellite

GPS device

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Satellite

3 packet message.

GPS device

Erasure Codes.

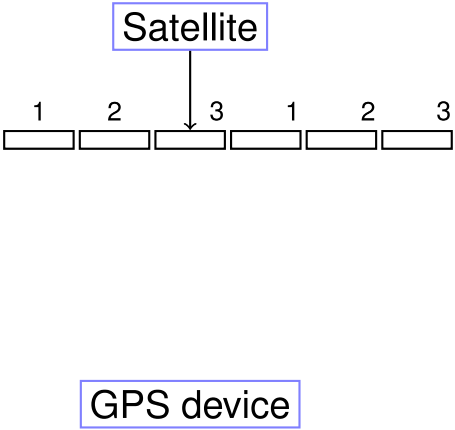
Satellite

3 packet message.

Lose 3 out 6 packets.

GPS device

Erasure Codes.



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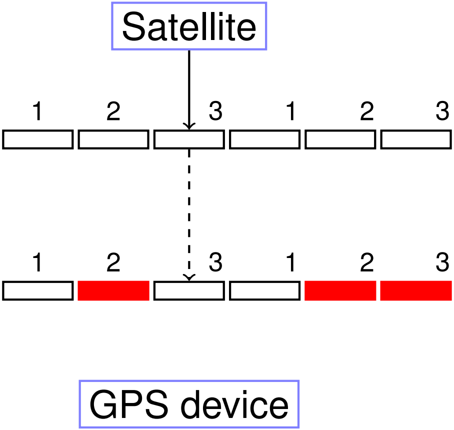

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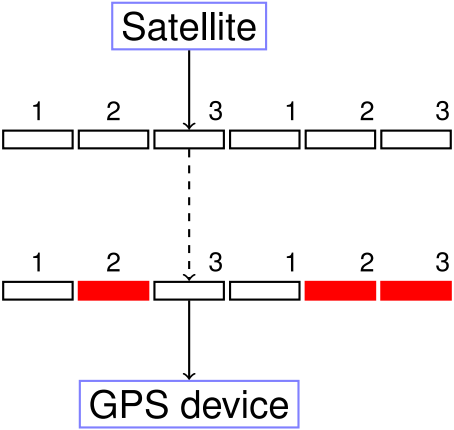

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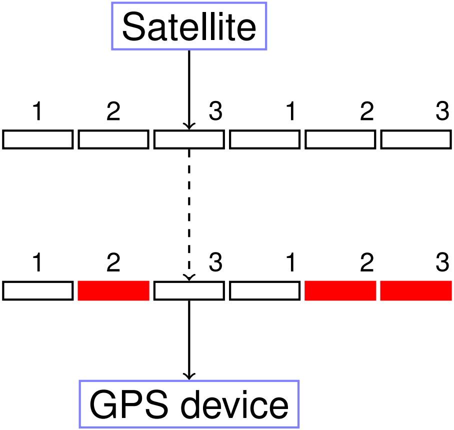

3 packet message. So send 6!

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Gets packets 1,1,and 3.

Solution Idea.

n packet message, channel that loses k packets.

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Must send $n+k$ packets!

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Representing vector (message) in different basis.

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Linear Algebra View:
Representing vector (message) in different basis.
Many bases!

The Scheme

Problem: Want to send a message with n packets.

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Erasure Coding Scheme: message = $m_0, m_1 \ldots, m_{n-1}$.

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Satellite

GPS device

Erasure Codes.

Satellite

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GPS device

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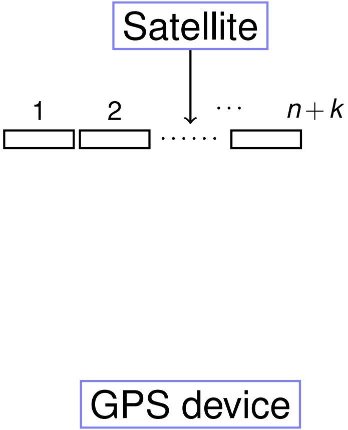
Satellite

n packet message.

Lose *k* packets.

GPS device

Erasure Codes.

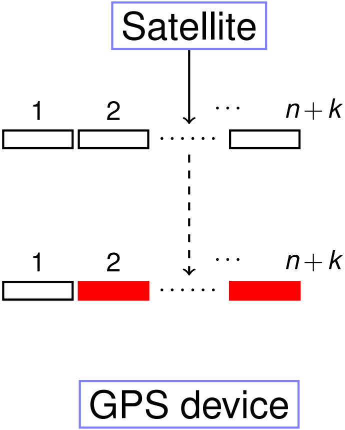


n packet message.

So send $n+k$ points on polynomial.

Lose k packets.

Erasure Codes.



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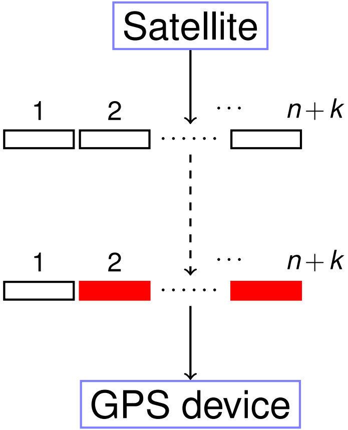

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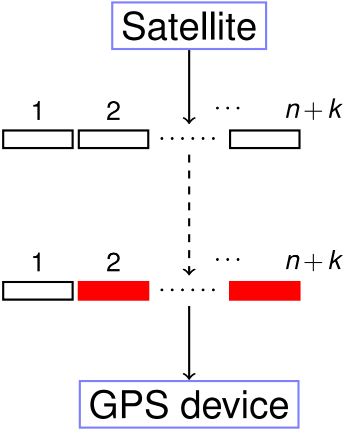

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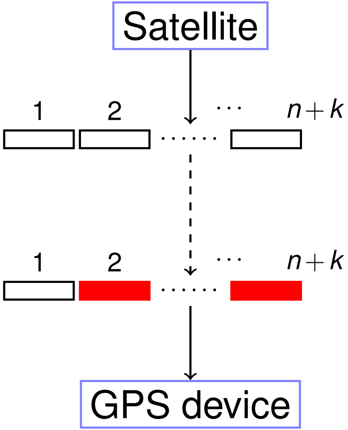

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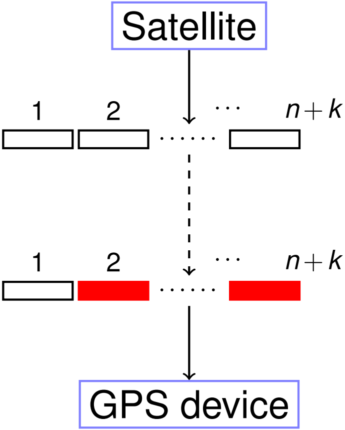

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Optimal.

Polynomials.

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- ..give Secret Sharing.

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Error Correction:

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Error Correction:

Noisy Channel: corrupts k packets. (rather than loses.)

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Noisy Channel: corrupts k packets. (rather than loses.)

Additional Challenge: Finding which packets are corrupt.

Error Correction

Satellite

GPS device

Which one was corrupted?

Error Correction

Satellite

3 packet message.

GPS device

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Error Correction

Satellite

3 packet message.

Corrupts 1 packets.

GPS device

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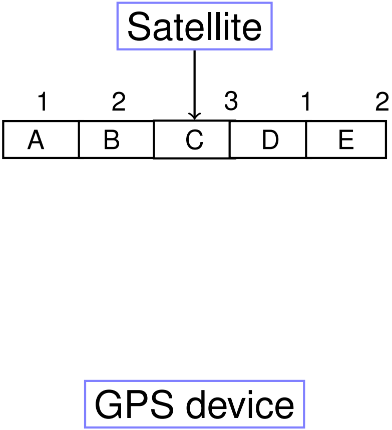
Error Correction



3 packet message. Send 5.

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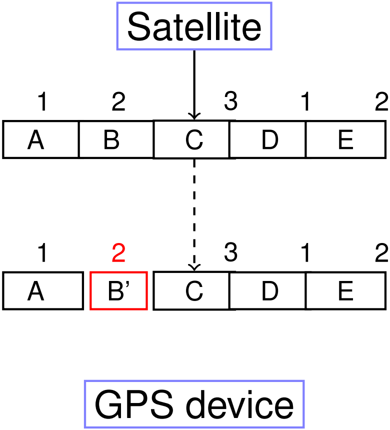
Error Correction



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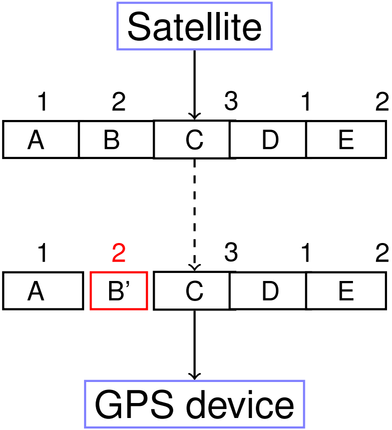

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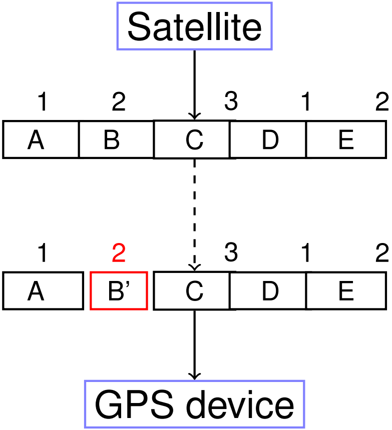

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The Scheme.

Problem: Communicate n packets $m_1, \ldots, m_n$ on noisy channel that corrupts $\leq k$ packets.

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After noisy channel: Recieve values $R(1), \ldots, R(n+2k)$.

Properties:
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Properties: proof.

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(1) $P(i) = R(i)$ for at least $n+k$ points i,
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that contains $\geq n+k$ received points.

Proof:

(1) Sure.

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There are only 5. So they disagree on 2.

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Degree 3 $\implies P(x) = Q(x)$

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$P(i) = R(i)$ for $n + k = 3 + 1 = 4$ points.

Slow solution.

Brute Force:
For each subset of $n+k$ points

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Reconstructs $P(x)$ and only $P(x)$!!

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Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

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All equations..

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\begin{aligned}
p_2 + p_1 + p_0 &\equiv 3 \pmod 7 \\
4p_2 + 2p_1 + p_0 &\equiv 1 \pmod 7 \\
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Assume point 2 is wrong and solve...consistent solution!

In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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$$\begin{aligned} p_{n-1} + \cdots p_0 &\equiv R(1) \pmod{p} \\ p_{n-1}2^{n-1} + \cdots p_0 &\equiv R(2) \pmod{p} \end{aligned}$$

In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

$$
\begin{aligned}
p_{n-1} + \cdots p_0 &\equiv R(1) \pmod{p} \\
p_{n-1}2^{n-1} + \cdots p_0 &\equiv R(2) \pmod{p} \\
&\cdot \\
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&\cdot \\
p_{n-1}(m)^{n-1} + \cdots p_0 &\equiv R(m) \pmod{p}
\end{aligned}
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&. \\
p_{n-1}(m)^{n-1} + \cdots p_0 &\equiv R(m) \pmod{p}
\end{aligned}$$

Error!!

In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n+2k)$.

$$
\begin{aligned}
p_{n-1} + \cdots p_0 &\equiv R(1) \pmod p \\
p_{n-1}2^{n-1} + \cdots p_0 &\equiv R(2) \pmod p \\
&. \\
p_{n-1}i^{n-1} + \cdots p_0 &\equiv R(i) \pmod p \\
&. \\
p_{n-1}(m)^{n-1} + \cdots p_0 &\equiv R(m) \pmod p
\end{aligned}
$$

Error!! .... Where???

In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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\begin{aligned}
p_{n-1} + \cdots p_0 &\equiv R(1) \pmod{p} \\
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&\cdot \\
p_{n-1}(m)^{n-1} + \cdots p_0 &\equiv R(m) \pmod{p}
\end{aligned}
$$

Error!! .... Where???
Could be anywhere!!!

In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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&\cdot \\
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\end{aligned}$$

Error!! .... Where???

Could be anywhere!!! ...so try everywhere.

In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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\end{aligned}
$$

Error!! .... Where???
Could be anywhere!!! ...so try everywhere.
Runtime: $\binom{n+2k}{k}$ possibilitities.

In general..

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Error!! .... Where???
Could be anywhere!!! ...so try everywhere.
Runtime: $\binom{n+2k}{k}$ possibilitities.

Something like $(n/k)^k$ ...Exponential in k!

In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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$$

Error!! .... Where???
Could be anywhere!!! ...so try everywhere.
Runtime: $\binom{n+2k}{k}$ possibilitities.

Something like $(n/k)^k$ ...Exponential in k!

How do we find where the bad packets are efficiently?!?!?!

Ditty...

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone..

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put
But the channel a bit wrong

Ditty...

Oh where, Oh where
has my little dog gone?
Oh where, oh where can he be

With his ears cut short
And his tail cut long
Oh where, oh where can he be?

Oh where, Oh where
have my packets gone.. wrong?
Oh where, oh where do they not fit.

With the polynomial well put
But the channel a bit wrong
Where, oh where do we look?

Where oh where can my bad packets be?

$$(p_{n-1} + \cdots p_0) \equiv R(1) \pmod{p}$$

Where oh where can my bad packets be?

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod p \\
(p_{n-1} 2^{n-1} + \cdots p_0) &\equiv R(2) \pmod p \\
&\vdots \\
(p_{n-1} (m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod p
\end{aligned}
$$

Where oh where can my bad packets be?

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod p \\
(p_{n-1} 2^{n-1} + \cdots p_0) &\equiv R(2) \pmod p \\
&\vdots \\
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\end{aligned}
$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Where oh where can my bad packets be?

$$\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
\mathbf{0} \times \quad (p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
&\vdots \\
(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}
\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

Where oh where can my bad packets be?

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But which equations should we multiply by 0?

Where oh where can my bad packets be?

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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!!

Where oh where can my bad packets be?

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\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1} 2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
&\vdots \\
(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}
\end{aligned}
$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!! That we don't know.

Where oh where can my bad packets be?

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1} 2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
&\vdots \\
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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!! That we don't know. But can find!

Where oh where can my bad packets be?

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
&\vdots \\
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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Where oh where can my bad packets be?

$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p}\\ (p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p}\\ &\vdots\\ (p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}\end{aligned}$$

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Error locator polynomial: $E(x) = (x - e_1)$

Where oh where can my bad packets be?

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p} \\
&\vdots \\
(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}
\end{aligned}
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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2)$

Where oh where can my bad packets be?

$$\begin{array}{rcll}
(p_{n-1} + \cdots p_0) & \equiv & R(1) & \pmod{p} \\
(p_{n-1}2^{n-1} + \cdots p_0) & \equiv & R(2) & \pmod{p} \\
& \vdots & & \\
(p_{n-1}(m)^{n-1} + \cdots p_0) & \equiv & R(n+2k) & \pmod{p}
\end{array}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2)\ldots$

Where oh where can my bad packets be?

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \qquad (\text{mod } p) \\
(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \qquad (\text{mod } p) \\
&\vdots \\
(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \qquad (\text{mod } p)
\end{aligned}
$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

We will use a polynomial!!! That we don't know. But can find!

Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2)\ldots(x - e_k)$.

Where oh where can my bad packets be?

$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod p \\ (p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod p \\ &\vdots \\ (p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod p\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2) \ldots (x - e_k)$.

$E(i) = 0$ if and only if $e_j = i$ for some j

Where oh where can my bad packets be?

$$\begin{aligned} E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\ E(2)(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2)E(2) \pmod{p} \\ &\vdots \\ E(m)(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k)E(m) \pmod{p} \end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

But which equations should we multiply by 0? Where oh where...??

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Multiply equations by $E(\cdot)$.

Where oh where can my bad packets be?

$$
\begin{aligned}
E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\
E(2)(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2)E(2) \pmod{p} \\
&\vdots \\
E(m)(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k)E(m) \pmod{p}
\end{aligned}
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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

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$E(i) = 0$ if and only if $e_j = i$ for some j

Multiply equations by $E(\cdot)$. (Above E(x) = (x-2).)

Where oh where can my bad packets be?

$$\begin{aligned}
E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\
E(2)(p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2)E(2) \pmod{p} \\
&\vdots \\
E(m)(p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k)E(m) \pmod{p}
\end{aligned}$$

Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.
All equations satisfied!!!!!

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Errors at points $e_1, \ldots, e_k$. (In diagram above, $e_1 = 2$.)

Error locator polynomial: $E(x) = (x - e_1)(x - e_2)\ldots(x - e_k)$.

$E(i) = 0$ if and only if $e_j = i$ for some j

Multiply equations by $E(\cdot)$. (Above E(x) = (x-2).)

All equations satisfied!!

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

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Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$
\begin{aligned}
(p_2 + p_1 + p_0) &\equiv (3) \pmod 7 \\
(4p_2 + 2p_1 + p_0) &\equiv (1) \pmod 7 \\
(2p_2 + 3p_1 + p_0) &\equiv (6) \pmod 7 \\
(2p_2 + 4p_1 + p_0) &\equiv (0) \pmod 7 \\
(4p_2 + 5p_1 + p_0) &\equiv (3) \pmod 7
\end{aligned}
$$

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

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(4p_2 + 5p_1 + p_0) &\equiv (3) \pmod 7
\end{aligned}
$$

Error locator polynomial: $(x - 2)$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$
\begin{aligned}
(1-2)(p_2 + p_1 + p_0) &\equiv (3)(1-2) \pmod 7 \\
(2-2)(4p_2 + 2p_1 + p_0) &\equiv (1)(2-2) \pmod 7 \\
(3-2)(2p_2 + 3p_1 + p_0) &\equiv (6)(3-2) \pmod 7 \\
(4-2)(2p_2 + 4p_1 + p_0) &\equiv (0)(4-2) \pmod 7 \\
(5-2)(4p_2 + 5p_1 + p_0) &\equiv (3)(5-2) \pmod 7
\end{aligned}
$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

Plugin points...

$$\begin{aligned}
(1-2)(p_2 + p_1 + p_0) &\equiv (3)(1-2) \pmod 7 \\
(2-2)(4p_2 + 2p_1 + p_0) &\equiv (1)(2-2) \pmod 7 \\
(3-2)(2p_2 + 3p_1 + p_0) &\equiv (6)(3-2) \pmod 7 \\
(4-2)(2p_2 + 4p_1 + p_0) &\equiv (0)(4-2) \pmod 7 \\
(5-2)(4p_2 + 5p_1 + p_0) &\equiv (3)(5-2) \pmod 7
\end{aligned}$$

Error locator polynomial: $(x - 2)$.

Multiply equation i by $(i - 2)$. All equations satisfied!

Example.

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Find $P(x) = p_2x^2 + p_1x + p_0$ that contains $n + k = 3 + 1$ points.

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(1-2)(p_2 + p_1 + p_0) &\equiv (3)(1-2) \pmod 7 \\
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4 unknowns (p_0, p_1, p_2 and e), 5 nonlinear equations.

..turn their heads each day,

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod p \\
&\vdots \\
(p_{n-1} i^{n-1} + \cdots p_0) &\equiv R(i) \pmod p \\
&\vdots \\
(p_{n-1}(n+2k)^{n-1} + \cdots p_0) &\equiv R(m) \pmod p
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\begin{aligned}
E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\
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Solving for $Q(x)$ and $E(x)$...

For all points $1, \ldots, i, n+2k = m$,

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$$\begin{aligned} a_3 + a_2 + a_1 + a_0 &\equiv 3(1 - b_0) \pmod 7 \\ a_3 + 4a_2 + 2a_1 + a_0 &\equiv 1(2 - b_0) \pmod 7 \end{aligned}$$

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

$Q(x) = E(x)P(x) = a_3x^3 + a_2x^2 + a_1x + a_0$

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$$
\begin{aligned}
a_3 + a_2 + a_1 + a_0 &\equiv 3(1 - b_0) \pmod 7 \\
a_3 + 4a_2 + 2a_1 + a_0 &\equiv 1(2 - b_0) \pmod 7 \\
6a_3 + 2a_2 + 3a_1 + a_0 &\equiv 6(3 - b_0) \pmod 7 \\
a_3 + 2a_2 + 4a_1 + a_0 &\equiv 0(4 - b_0) \pmod 7 \\
6a_3 + 4a_2 + 5a_1 + a_0 &\equiv 3(5 - b_0) \pmod 7
\end{aligned}
$$

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6a_3 + 4a_2 + 5a_1 + a_0 &\equiv 3(5 - b_0) \pmod 7
\end{aligned}$$

$a_3 = 1$, $a_2 = 6$, $a_1 = 6$, $a_0 = 5$ and $b_0 = 2$.

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\end{aligned}$$

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$Q(x) = x^3 + 6x^2 + 6x + 5$.

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6a_3 + 4a_2 + 5a_1 + a_0 &\equiv 3(5 - b_0) \pmod 7
\end{aligned}$$

$a_3 = 1$, $a_2 = 6$, $a_1 = 6$, $a_0 = 5$ and $b_0 = 2$.

$Q(x) = x^3 + 6x^2 + 6x + 5$.

$E(x) = x - 2$.

Example: finishing up.

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$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
           __________________
x - 2 ) x^3  + 6 x^2 + 6 x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2
            -----------------
x - 2 )  x^3   + 6 x^2 + 6 x + 5
         x^3   - 2 x^2
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2
         -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                  1 x^2 + 1 x
         ----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                  1 x^2 + 6 x + 5
                  1 x^2 - 2 x
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x
          -----------------
 x - 2 )  x^3  + 6 x^2 + 6 x + 5
          x^3  - 2 x^2
          ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                1 x^2 + 1 x + 1
         -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
                ---------------
                          x + 5
                          x - 2
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
        -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
               1 x^2 + 6 x + 5
               1 x^2 - 2 x
               ---------------
                         x + 5
                         x - 2
                         -----
                             0
```

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

$P(x) = x^2 + x + 1$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                   1 x^2 + 1 x + 1
          -----------------
 x - 2 )  x^3  + 6 x^2 + 6 x + 5
          x^3  - 2 x^2
          ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                         x + 5
                         x - 2
                         -----
                             0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$?

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                1 x^2 + 1 x + 1
          -----------------
x - 2 )   x^3  + 6 x^2 + 6 x + 5
          x^3  - 2 x^2
          ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
                ---------------
                          x + 5
                          x - 2
                          -----
                              0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
          ----------------- 
x - 2 )  x^3   + 6 x^2 + 6 x + 5
         x^3   - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1
Except at $x = 2$?

Example: finishing up.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
           ----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6.$

What is $\frac{x-2}{x-2}$? 1
Except at $x = 2$? Hole there?

Error Correction: Berlekamp-Welsh

Message: $m_1, \ldots, m_n$.
Sender:

1. Form degree $n-1$ polynomial $P(x)$ where $P(i) = m_i$.
2. Send $P(1), \ldots, P(n+2k)$.

Receiver:

1. Receive $R(1), \ldots, R(n+2k)$.
2. Solve $n+2k$ equations, $Q(i) = E(i)R(i)$ to find $Q(x) = E(x)P(x)$ and $E(x)$.
3. Compute $P(x) = Q(x)/E(x)$.
4. Compute $P(1), \ldots, P(n)$.

Check your undersanding.

You have error locator polynomial!

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Check your undersanding.

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Where oh where have my packets gone wrong?

Factor?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency?

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+2k$ values.

Check your undersanding.

You have error locator polynomial!

Where oh where have my packets gone wrong?

Factor? Sure.
Check all values? Sure.

Efficiency? Sure. Only $n+2k$ values.
See where it is 0.

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Hmmm...

Is there one and only one $P(x)$ from Berlekamp-Welsh procedure?

Existence: there is a $P(x)$ and $E(x)$ that satisfy equations.

Unique solution for $P(x)$

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \tag{1}$$

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Proof:

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Proof:
We claim

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Proof:
We claim

$$Q'(x)E(x) = Q(x)E'(x) \text{ on } n+2k \text{ values of } x. \tag{2}$$

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Equation 2 implies 1:

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$Q'(x)E(x)$ and $Q(x)E'(x)$ are degree $n+2k-1$

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Equation 2 implies 1:

$Q'(x)E(x)$ and $Q(x)E'(x)$ are degree $n+2k-1$
and agree on $n+2k$ points

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Equation 2 implies 1:

$Q'(x)E(x)$ and $Q(x)E'(x)$ are degree $n+2k-1$
and agree on $n+2k$ points
$E(x)$ and $E'(x)$ have at most k zeros each.

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Can cross divide at n points.

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$\implies \frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)}$ equal on n points.

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Both degree $\leq n$

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Can cross divide at n points.
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Both degree $\leq n \implies$ Same polynomial!

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$\implies \frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)}$ equal on n points.
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Last bit.

Fact: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

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Proof: Construction implies that

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Proof: Construction implies that

$$Q(i) = R(i)E(i)$$
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for $i \in \{1, \ldots n+2k\}$.

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$$Q(i) = R(i)E(i)$$
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If $E(i) = 0$, then $Q(i) = 0$.

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$$Q(i) = R(i)E(i)$$

$$Q'(i) = R(i)E'(i)$$

for $i \in \{1, \ldots n+2k\}$.

If $E(i) = 0$, then $Q(i) = 0$. If $E'(i) = 0$, then $Q'(i) = 0$.
$\implies Q(i)E'(i) = Q'(i)E(i)$ holds when $E(i)$ or $E'(i)$ are zero.

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When $E'(i)$ and $E(i)$ are not zero

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When $E'(i)$ and $E(i)$ are not zero

$$\frac{Q'(i)}{E'(i)} = \frac{Q(i)}{E(i)} = R(i).$$

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When $E'(i)$ and $E(i)$ are not zero

$$\frac{Q'(i)}{E'(i)} = \frac{Q(i)}{E(i)} = R(i).$$

Cross multiplying gives equality in fact for these points.

Last bit.

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for $i \in \{1, \dots n+2k\}$.

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When $E'(i)$ and $E(i)$ are not zero

$$\frac{Q'(i)}{E'(i)} = \frac{Q(i)}{E(i)} = R(i).$$

Cross multiplying gives equality in fact for these points. □

Points to polynomials, have to deal with zeros!

Last bit.

Fact: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

Proof: Construction implies that

$$Q(i) = R(i)E(i)$$
$$Q'(i) = R(i)E'(i)$$

for $i \in \{1, \ldots n+2k\}$.

If $E(i) = 0$, then $Q(i) = 0$. If $E'(i) = 0$, then $Q'(i) = 0$.
$\implies Q(i)E'(i) = Q'(i)E(i)$ holds when $E(i)$ or $E'(i)$ are zero.

When $E'(i)$ and $E(i)$ are not zero

$$\frac{Q'(i)}{E'(i)} = \frac{Q(i)}{E(i)} = R(i).$$

Cross multiplying gives equality in fact for these points. □

Points to polynomials, have to deal with zeros!

Example: dealing with $\frac{x-2}{x-2}$ at $x = 2$.

Yaay!!

Berlekamp-Welsh algorithm decodes correctly when k errors!

Summary. Error Correction.

Communicate n packets, with k erasures.

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Reed-Solomon codes.

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Reed-Solomon codes. Welsh-Berlekamp Decoding.

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Reed-Solomon codes. Welsh-Berlekamp Decoding. Perfection!

Wow.

Lots of material today...