Today.

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Polynomials.

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Secret Sharing.

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Secret Sharing.

Correcting for loss or even corruption.

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Two points make a line.

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Secrecy: Any $k-1$ knows nothing.
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Efficient: minimize storage.

The idea of the day.

Two points make a line.
Lots of lines go through one point.

Polynomials

A **polynomial**

$$P(x) = a_d x^d + a_{d-1} x^{d-1} \cdots + a_0.$$

is specified by **coefficients** $a_d, \ldots a_0$.

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[1] A field is a set of elements with addition and multiplication operations, with inverses. $GF(p) = (\{0, \ldots, p-1\}, + \pmod p, * \pmod p)$.

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Polynomials $P(x)$ with arithmetic modulo p: [1] $a_i \in \{0, \ldots, p-1\}$ and

$$P(x) = a_d x^d + a_{d-1} x^{d-1} \cdots + a_0 \pmod{p},$$

for $x \in \{0, \ldots, p-1\}$.

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Polynomial: $P(x) = a_d x^4 + \cdots + a_0$

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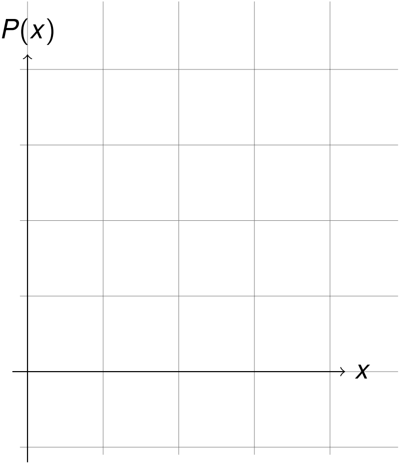

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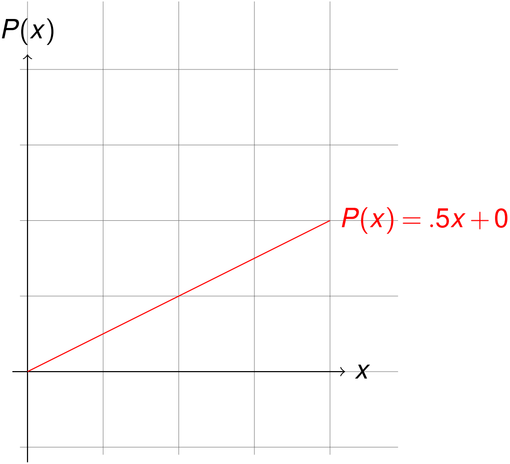

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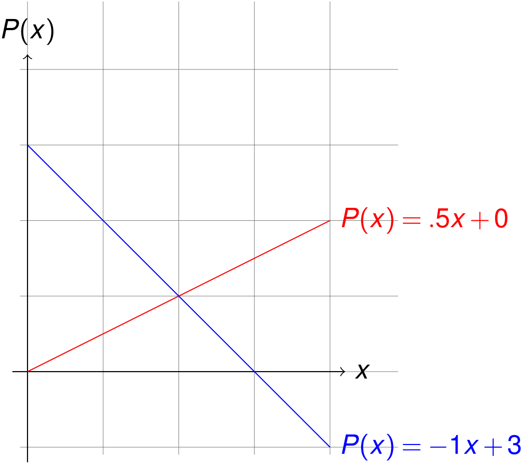

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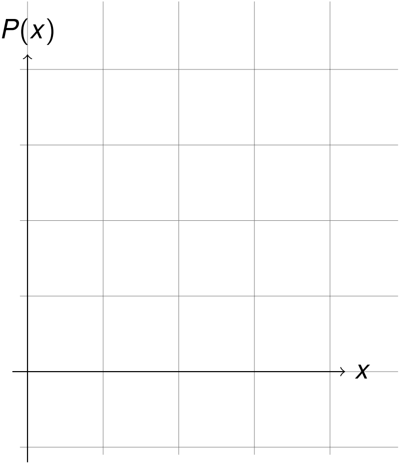

Parabola: $P(x) = a_2 x^2 + a_1 x + a_0$

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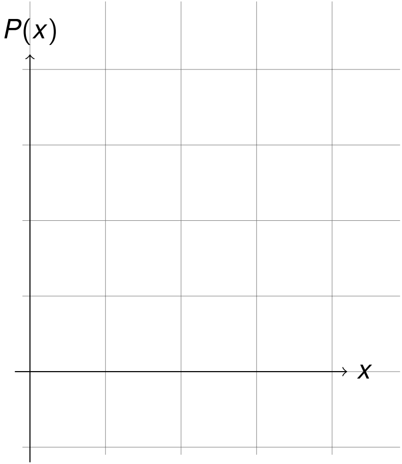

Parabola: $P(x) = a_2 x^2 + a_1 x + a_0 = ax^2 + bx + c$

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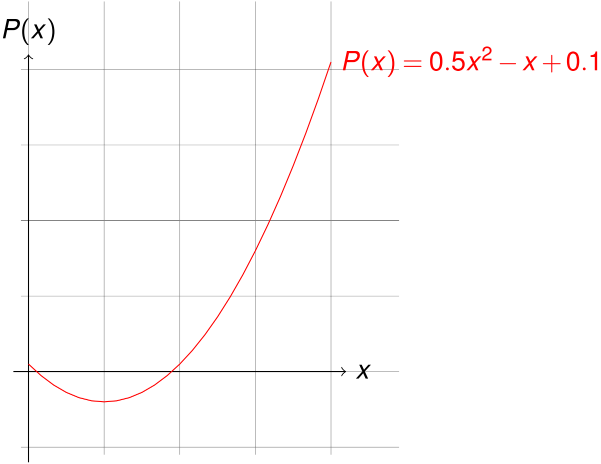

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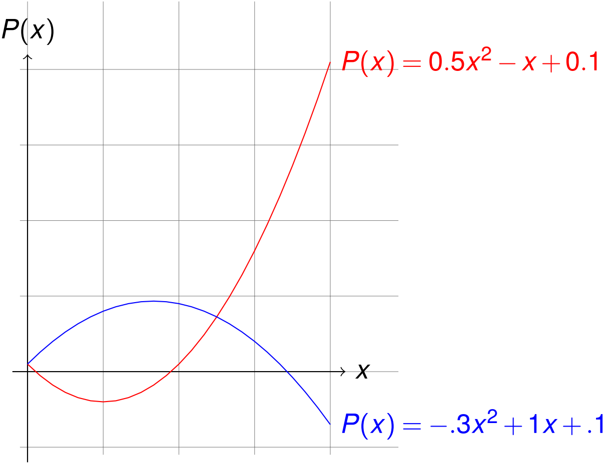

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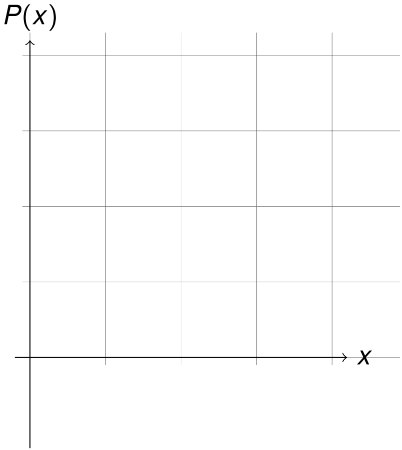

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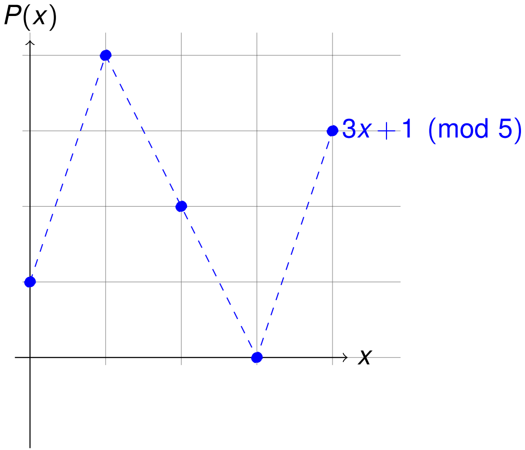

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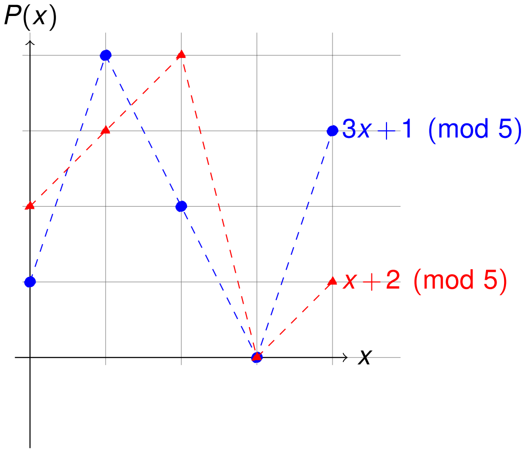

Finding an intersection.
$x + 2 \equiv 3x + 1 \pmod 5$
$\implies 2x \equiv 1 \pmod 5$

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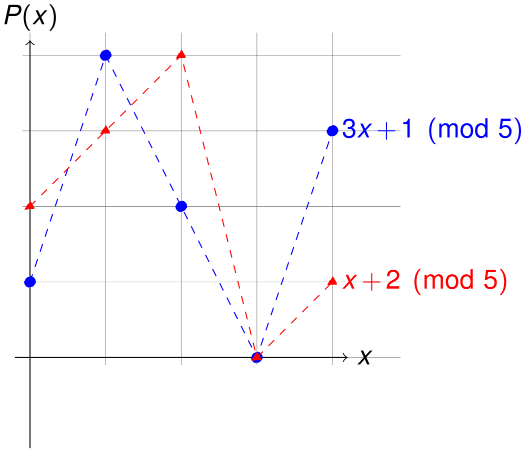

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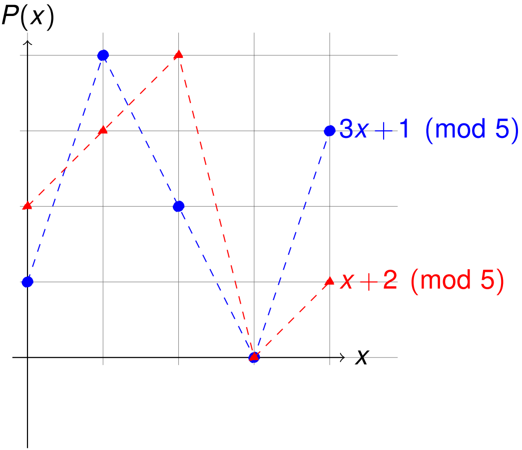

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Good when modulus is prime!!

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Fact: Exactly 1 degree $\leq d$ polynomial contains $d+1$ points. [2]

[2] Points with different x values.

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Modular Arithmetic Fact: Exactly 1 degree $\leq d$ polynomial with arithmetic modulo prime p contains $d+1$ pts.

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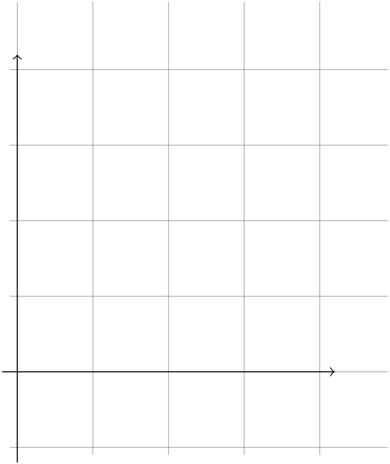
3 points determine a parabola.



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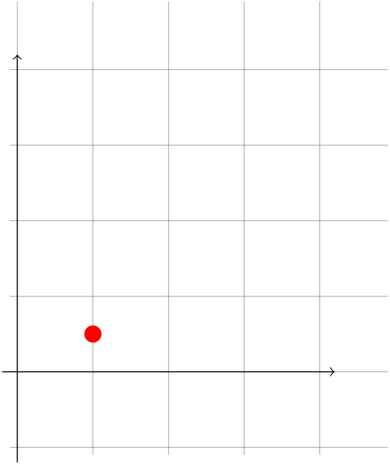

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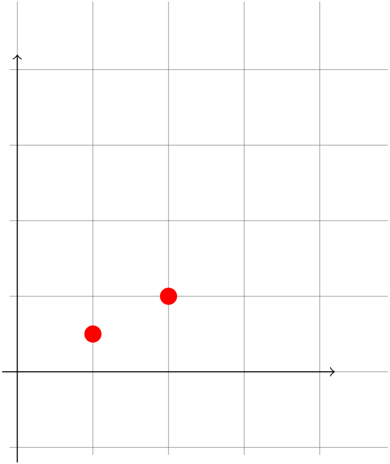

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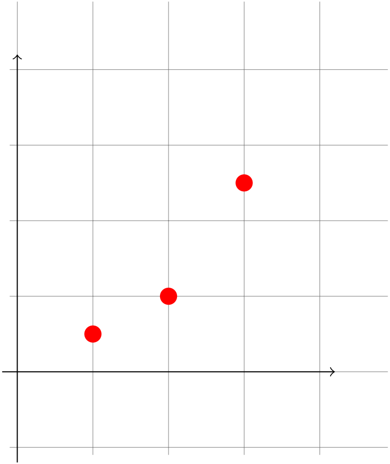

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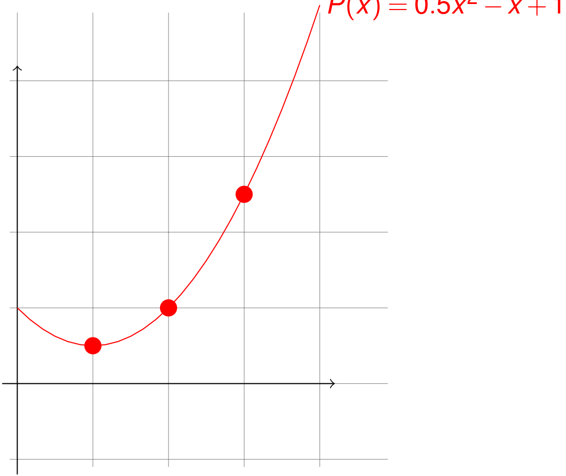

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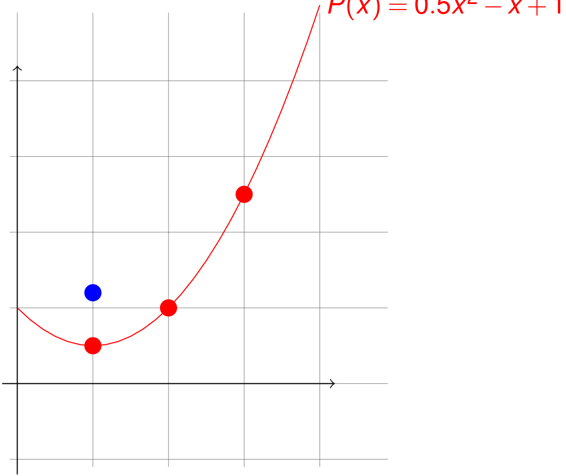

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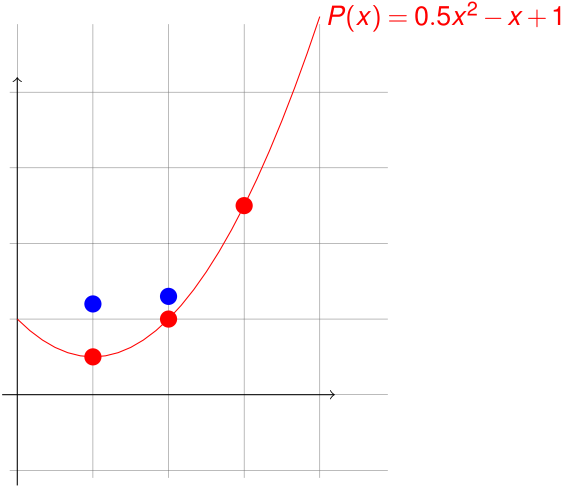

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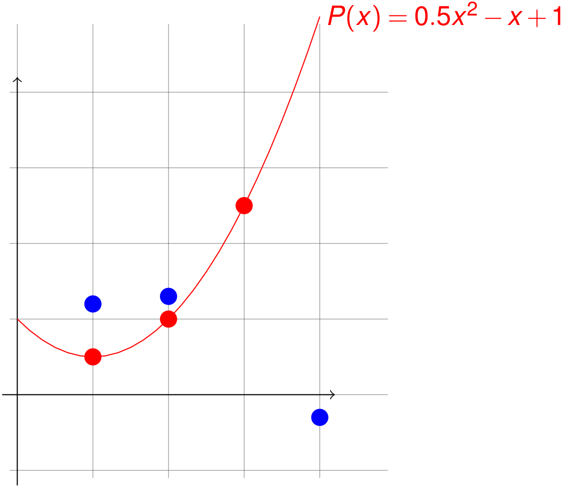

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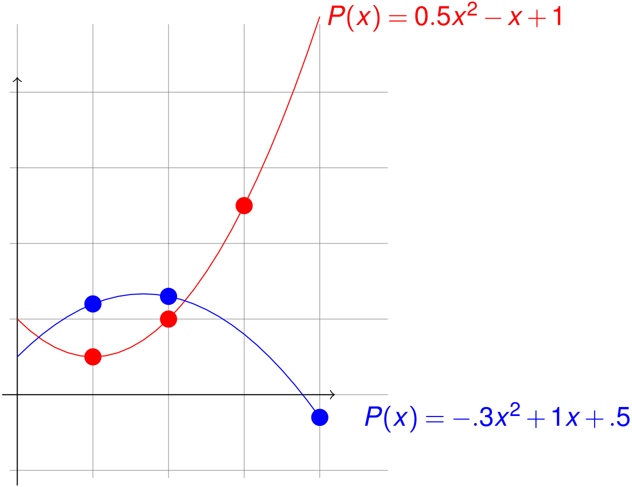

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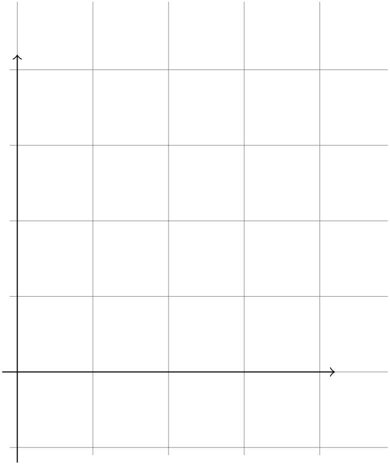

There is $P(x)$ contains blue points and *any* $(0, y)$!

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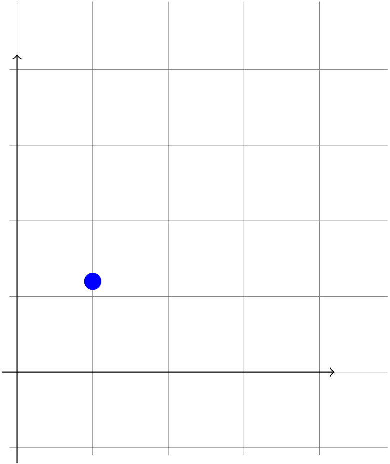

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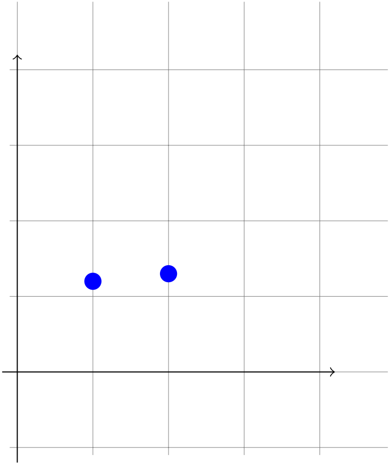

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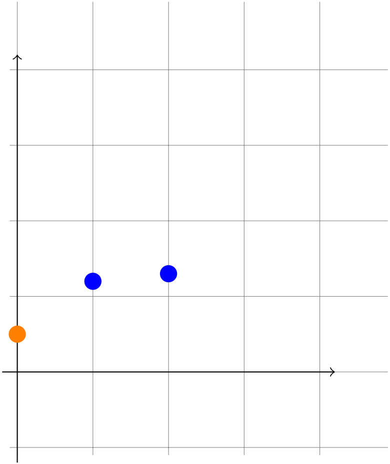

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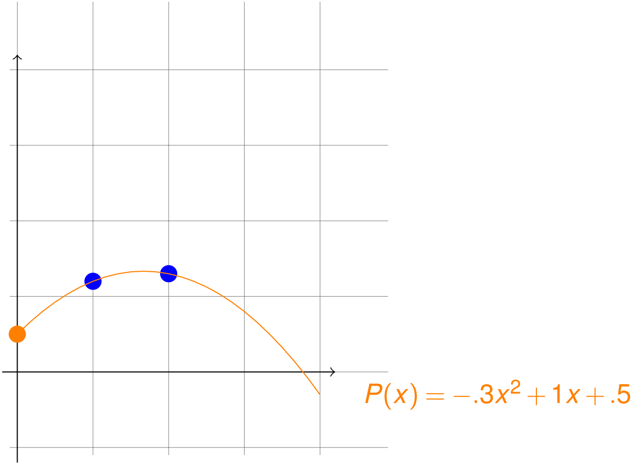

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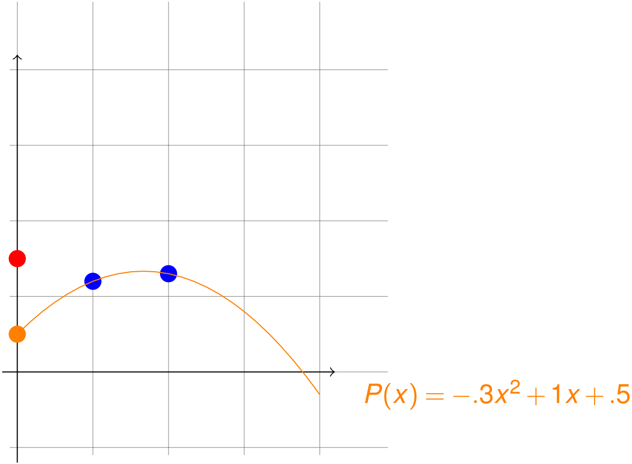

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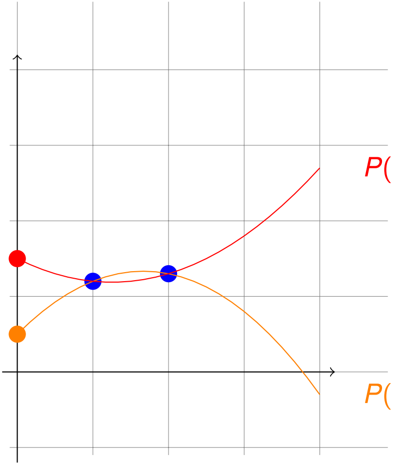

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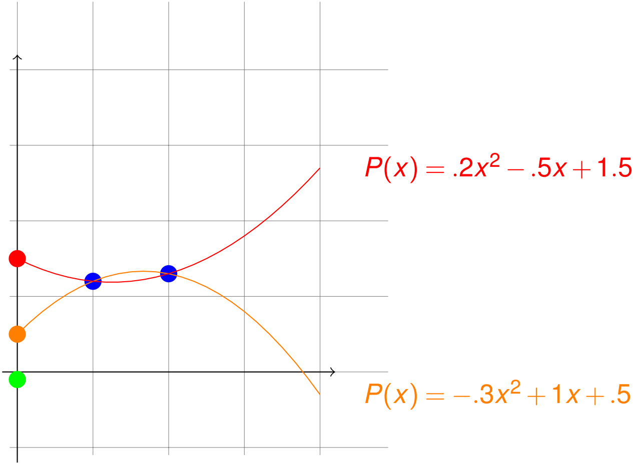

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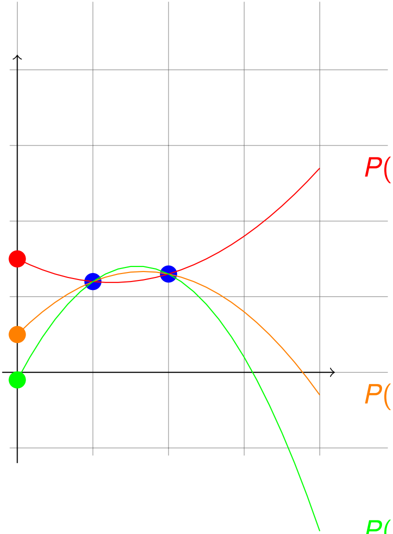

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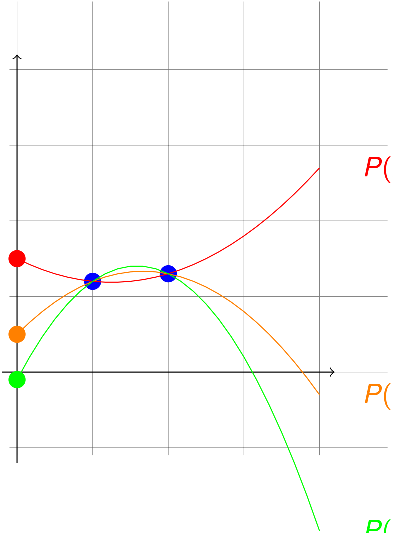

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Knowing k pts $\implies$ only one $P(x)$ $\implies$ evaluate $P(0)$.
Secrecy: Any $k-1$ shares give nothing.

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Multiplicative inverses due to $gcd(x,p) = 1$, forall $x \in \{1,\ldots,p-1\}$

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Construction proves the existence of the polynomial!

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Must prove **Roots fact.**

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That is, $P(x) = (x - a)Q(x) + r$

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Lemma 1: $P(x)$ has root a iff $P(x)/(x-a)$ has remainder 0: $P(x) = (x-a)Q(x)$.

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Intuitively, a field is a set with operations corresponding to addition, multiplication, and division.

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Modular Arithmetic Fact: Exactly one polynomial degree $\leq d$ over $GF(p)$, $P(x)$, that hits $d+1$ points.

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Roubustness: Any k knows secret.
Knowing k pts, only one $P(x)$, evaluate $P(0)$.
Secrecy: Any $k-1$ knows nothing.

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1. Choose $a_0 = s$, and randomly $a_1, \ldots, a_{k-1}$.
2. Let $P(x) = a_{k-1}x^{k-1} + a_{k-2}x^{k-2} + \cdots a_0$ with $a_0 = s$.
3. Share i is point $(i, P(i) \mod p)$.

Roubustness: Any k knows secret.
Knowing k pts, only one $P(x)$, evaluate $P(0)$.
Secrecy: Any $k-1$ knows nothing.
Knowing $\leq k-1$ pts, any $P(0)$ is possible.

Secret Sharing

Modular Arithmetic Fact: Exactly one polynomial degree $\leq d$ over $GF(p)$, $P(x)$, that hits $d+1$ points.

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(Almost) the same as what is missing: one $P(i)$.

Runtime.

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Runtime: polynomial in k, n, and $\log p$.

1. Evaluate degree $k-1$ polynomial n times using $\log p$-bit numbers.
2. Reconstruct secret by solving system of k equations using $\log p$-bit arithmetic.

A bit more counting.

What is the number of degree d polynomials over $GF(m)$?

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Infinite number for reals, rationals, complex numbers!

Erasure Codes.

Satellite

GPS device

Erasure Codes.

Satellite

3 packet message.

GPS device

Erasure Codes.

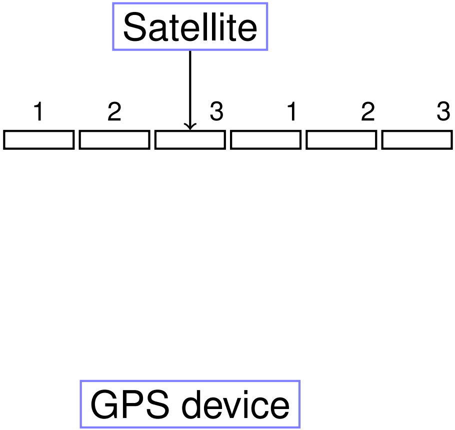
Satellite

3 packet message.

Lose 3 out 6 packets.

GPS device

Erasure Codes.



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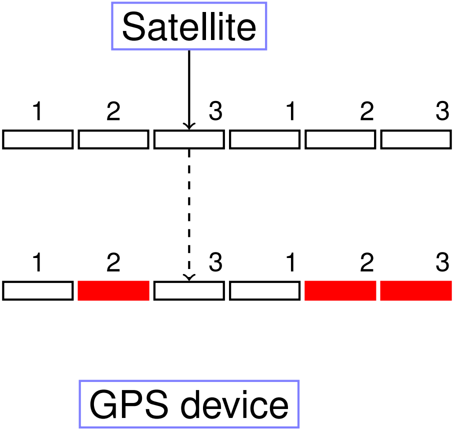

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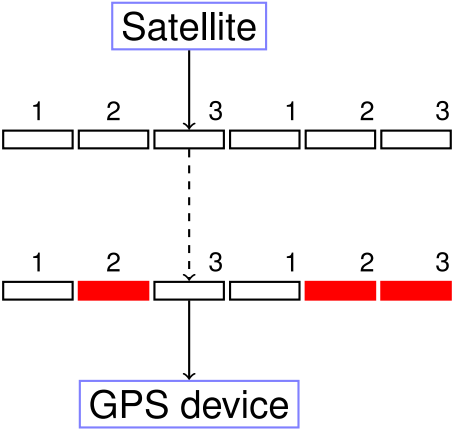

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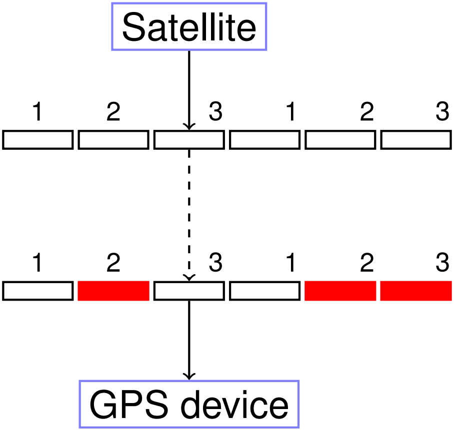

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Gets packets 1,1,and 3.

Solution Idea.

n packet message, channel that loses *k* packets.

Solution Idea.

n packet message, channel that loses k packets.

Must send $n+k$ packets!

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Any n packets

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Use polynomials.

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Problem: Want to send a message with n packets.

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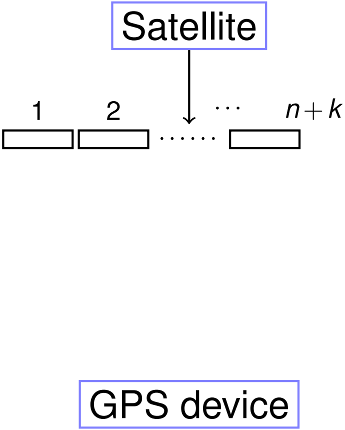

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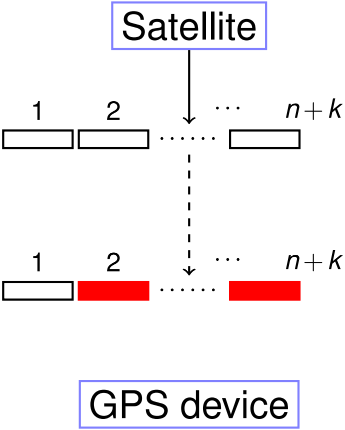

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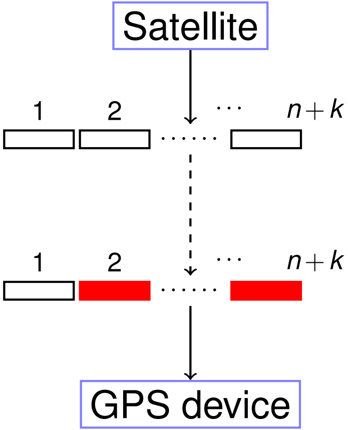

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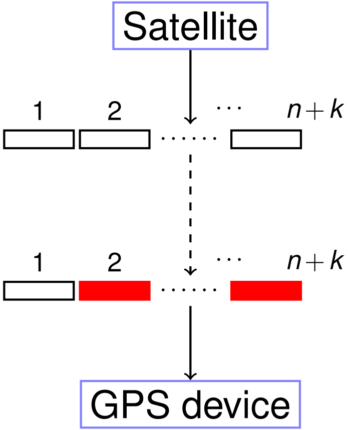

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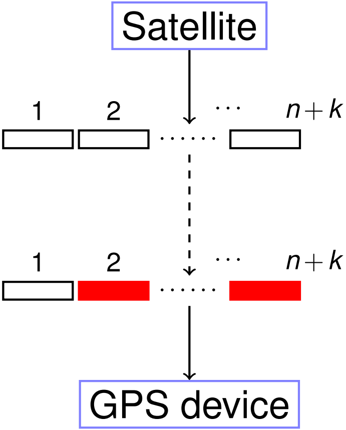

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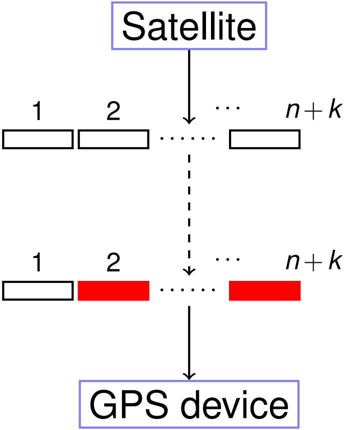

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$$P(1) = a_2 + a_1 + a_0 \quad \equiv \quad 1 \pmod 7$$

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$6a_1 + 3a_0 = 2 \pmod{7}$,

Example

Make polynomial with $P(1) = 1$, $P(2) = 4$, $P(3) = 4$.

Modulo 7 to accommodate at least 6 packets.

Linear equations:

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Packets: $(1,1),(2,4),(3,4),(4,7),(5,2),(6,0)$

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Notice that packets contain “x-values”.

Bad reception!

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Channeling Sahai

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$P(x) = 2x^2 + 4x + 2$

Message? $P(1) = 1,$

Bad reception!

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Message? $P(1) = 1, P(2) = 4, P(3) = 4$.

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You want to encode a secret consisting of 1,4,4.

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through a noisy channel that loses 3 packets.

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How big should modulus be?
Larger than 8

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Send n packets b-bit packets, with k errors.

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Send n packets b-bit packets, with k errors.
Modulus should be larger than $n + k$ and also larger than 2^b.

Polynomials.

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- ..give Secret Sharing.

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- ..give Secret Sharing.
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Noisy Channel: corrupts k packets. (rather than loss.)

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Additional Challenge: Finding which packets are corrupt.

Error Correction

Satellite

GPS device

Error Correction

Satellite

3 packet message.

GPS device

Error Correction

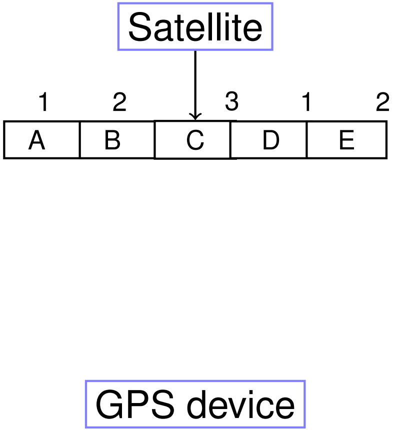
Satellite

3 packet message.

Corrupts 1 packets.

GPS device

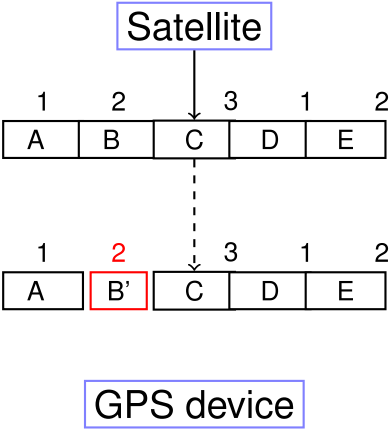
Error Correction



3 packet message. Send 5.

Corrupts 1 packets.

Error Correction



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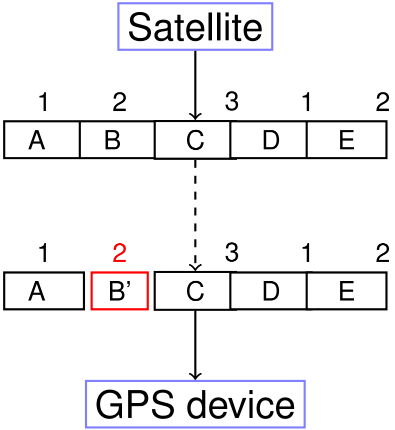

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The Scheme.

Problem: Communicate n packets $m_1, \ldots, m_n$
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After noisy channel: Recieve values $R(1), \ldots, R(n+2k)$.

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$P(i) = R(i)$ for $n + k = 3 + 1 = 4$ points.

Slow solution.

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For each subset of $n+k$ points

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Reconstructs $P(x)$ and only $P(x)$!!

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Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

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All equations..

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$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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$$p_{n-1} + \cdots p_0 \equiv R(1) \pmod{p}$$

In general..

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How do we find where the bad packets are efficiently?!?!?!

Ditty...

Ditty...

Where oh where

Ditty...

Where oh where can my bad packets be ...

Ditty...

Where oh where can my bad packets be ...

Ditty...

Where oh where can my bad packets be ...

On Thursday.