70: Discrete Math and Probability Theory

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See note 1, for more discussion.

Babak Ayazifar

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Does time in 517 Cory Hall. Make appointment before knocking.

Satish Rao

19th year at Berkeley.

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PhD: Long time ago,

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Announcements, logistics, critical advice.

Wason's experiment:1

Suppose we have four cards on a table:

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- Suppose you see that Alice went to Baltimore, Bob drove,
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Alice	Bob	Charlie	Donna
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Answer:

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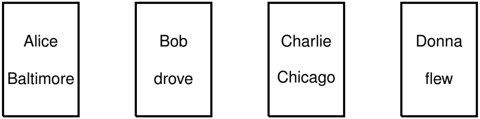

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Answer: Later.

CS70: Lecture 1. Outline.

Today: Note 1.

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The language of proofs!

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The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational
2+2 = 4
2+2 = 3
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“$2+2=3$” $\bigwedge$ “$2+2=4$” – a proposition that is ... False

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True when both P and Q are True . Else False .

Disjunction ("or"): $P \vee Q$

"$P \vee Q$" is True when at least one P or Q is True . Else False .

Negation ("not"): $\neg P$

"$\neg P$" is True when P is False . Else False .

Examples:

$\neg$ "$(2+2=4)$" – a proposition that is ... False

"$2+2=3$" $\bigwedge$ "$2+2=4$" – a proposition that is ... False

"$2+2=3$" $\bigvee$ "$2+2=4$" – a proposition that is ...

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Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

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$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...False .

Propositional Forms: quick check!

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Propositional Forms: quick check!

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$P \wedge Q$ ...

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
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Propositional Forms: quick check!

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$Q =$ "826th digit of pi is 2"

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Q is ...True .

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$P \vee Q$ ... True

$\neg P$ ...

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...False .
Q is ...True .

$P \wedge Q$ ... False

$P \vee Q$ ... True

$\neg P$ ... True

Put them together..

Propositions:

P_1 - Person 1 rides the bus.

Put them together..

Propositions:

P_1 - Person 1 rides the bus.

P_2 - Person 2 rides the bus.

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Propositions:

P_1 - Person 1 rides the bus.

P_2 - Person 2 rides the bus.

....

Put them together..

Propositions:
P_1 - Person 1 rides the bus.
P_2 - Person 2 rides the bus.
....

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

Put them together..

Propositions:

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Propositional Form:

$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

Put them together..

Propositions:
P_1 - Person 1 rides the bus.
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Propositional Form:
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Can person 3 ride the bus?

Put them together..

Propositions:
P_1 - Person 1 rides the bus.
P_2 - Person 2 rides the bus.
....

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

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$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

Put them together..

Propositions:
P_1 - Person 1 rides the bus.
P_2 - Person 2 rides the bus.
....

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

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Can person 3 and person 4 ride the bus together?

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Propositions:
P_1 - Person 1 rides the bus.
P_2 - Person 2 rides the bus.
....

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

Propositional Form:
$\neg(((P_1 \lor P_2) \land (P_3 \lor P_4)) \lor ((P_2 \lor P_3) \land (P_4 \lor \neg P_5)))$

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

This seems ...

Put them together..

Propositions:

P_1 - Person 1 rides the bus.

P_2 - Person 2 rides the bus.

....

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Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

This seems ...complicated.

We can program!!!!

Put them together..

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Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

This seems ...complicated.

We can program!!!!

We need a way to keep track!

Truth Tables for Propositional Forms.

P	Q	$P \wedge Q$
T	T	T
T	F	
F	T	
F	F	

Truth Tables for Propositional Forms.

P	Q	$P \wedge Q$
T	T	T
T	F	F
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Notice: $\wedge$ and $\vee$ are commutative.

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One use for truth tables: Logical Equivalence of propositional forms!

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One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$

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F	F	F

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T	T	F	F
T	F	F	F
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Truth Tables for Propositional Forms.

P	Q	$P \wedge Q$
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F	T	F
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Notice: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$

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P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q)$

Truth Tables for Propositional Forms.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
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T	F	T
F	T	T
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Notice: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$

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P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$$\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q$$

Truth Tables for Propositional Forms.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

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DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q \qquad\qquad \neg(P \vee Q)$

Truth Tables for Propositional Forms.

P	Q	$P \wedge Q$
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P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q \qquad\qquad \neg(P \vee Q) \quad \equiv \quad \neg P \wedge \neg Q$

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	T

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Is $(T \wedge Q) \equiv Q$?

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	T

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes?

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	T

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	T

Is $(T \wedge Q) \equiv Q$? Yes? No?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	T

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes!

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	T

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
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F	F	T

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
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F	F	T

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

Quick Questions

P	Q	$P \wedge Q$
T	T	T
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P	Q	$P \vee Q$
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
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P	Q	$P \vee Q$
T	T	T
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

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What is $(F \vee Q)$?

Quick Questions

P	Q	$P \wedge Q$
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P	Q	$P \vee Q$
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

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What is $(F \vee Q)$? Q

Distributive?

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)?$$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$,

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$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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Cases:

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RHS: $(T \wedge Q) \vee (T \wedge R)$

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LHS: $F \wedge (Q \vee R)$

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LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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P is True .

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Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

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LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

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P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$,

Distributive?

$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$?

Simplify: $(T \land Q) \equiv Q$, $(F \land Q) \equiv F$.

Cases:

P is True .

LHS: $T \land (Q \lor R) \equiv (Q \lor R)$.

RHS: $(T \land Q) \lor (T \land R) \equiv (Q \lor R)$.

P is False .

LHS: $F \land (Q \lor R) \equiv F$.

RHS: $(F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F$.

$P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)$?

Simplify: $T \lor Q \equiv T$, $F \lor Q \equiv Q$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Foil 1:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
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P is False .
LHS: $F \wedge (Q \vee R) \equiv F$.
RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Foil 1:
$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Foil 2:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

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$(A \wedge B) \vee (C \wedge D) \equiv (A \vee C) \wedge (A \vee D) \wedge (B \vee C) \wedge (B \vee D)$?

Implication.

$P \implies Q$ interpreted as

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If P, then Q.

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True Statements: P, $P \implies Q$.

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Examples:

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Examples:

Statement: If you stand in the rain, then you'll get wet.

Implication.

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Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"

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Examples:

Statement: If you stand in the rain, then you'll get wet.
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Examples:

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Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
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Statement:
If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

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Statement: If you stand in the rain, then you'll get wet.
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If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

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P = "a right triangle has sidelengths $a \leq b \leq c$",
Q = "$a^2 + b^2 = c^2$".

Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

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False implies nothing

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P False means Q can be True or False

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If chemical plant pollutes river, fish die.

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Some Fun: use propositional formulas to describe implication?

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Some Fun: use propositional formulas to describe implication?
$((P \implies Q) \wedge P) \implies Q$.

Implication and English.

$P \implies Q$

- If P, then Q.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
  Just reversing the order.

Implication and English.

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Implication and English.

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- If P, then Q.
- Q if P.
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- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.

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  This means that proving P allows you
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- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.
- P is sufficient for Q.
  This means that proving P allows you
  to conclude that Q is true.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	
F	T	
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
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Truth Table: implication.

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T	T	T
T	F	F
F	T	T
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P	Q	$\neg P \lor Q$
T	T	
T	F	
F	T	
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
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T	F	
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Truth Table: implication.

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T	T	T
T	F	F
F	T	T
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T	T	T
T	F	F
F	T	
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
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Truth Table: implication.

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These two propositional forms are logically equivalent!

Contrapositive, Converse

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- **Definition:** If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.
  (Logically Equivalent: $\iff$.)

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Next: Statements about boolean valued functions!!

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- $\mathbb{Z}^+$ (positive integers)

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- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).
- $\mathbb{Z} = \{\ldots, -1, 0, \ldots\}$ (integers)
- $\mathbb{Z}^+$ (positive integers)
- $\mathbb{R}$ (real numbers)

Quantifiers: universes.

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- See note 0 for more!

Back to: Wason's experiment:1

Theory:

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Statement/theory: $\forall x \in \{A, B, C, D\}, P(x)$

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$P(A) =$ False .

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No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

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Yes.

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Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.

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$Q(B)$ = False . Do we care about $P(B)$?
Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.
So $P(Bob)$ must be False .

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$P(C)$ = True .

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$P(C)$ = True . Do we care about $Q(C)$?

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So $P(Bob)$ must be False .

$P(C)$ = True . Do we care about $Q(C)$?
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So $P(Bob)$ must be False .

$P(C)$ = True . Do we care about $Q(C)$?
Yes. $P(C) \implies Q(C)$ means $Q(C)$ must be true.

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Only have to turn over cards for Bob and Charlie.

More for all quantifiers examples.

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- "Square of any natural number greater than 5 is greater than 25."

More for all quantifiers examples.

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Idea alert:

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Idea alert: Restrict domain using implication.

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Idea alert: Restrict domain using implication.

Later we may omit universe if clear from context.

Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

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Quantifiers....negation...DeMorgan again.

Consider

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English: there is an x in S where $P(x)$ does not hold.

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$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

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What we do in this course! We consider claims.

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What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$

Quantifiers....negation...DeMorgan again.

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Next Time: proofs!