# 1 Uniform Probability Space

Let $\Omega = \{1,2,3,4,5,6\}$ be a uniform probability space. Let also $X(\omega)$ and $Y(\omega)$, for $\omega \in \Omega$, be the random variables defined in the table:

Table 1: All the rows in the table correspond to random variables.

| $\omega$ | 1 | 2 | 3 | 4 | 5 | 6 | $\mathbb{E}[\cdot]$ |
|---|---|---|---|---|---|---|---|
| $X(\omega)$ | 0 | 0 | 1 | 1 | 2 | 2 | |
| $Y(\omega)$ | 0 | 2 | 3 | 5 | 2 | 0 | |
| $X^2(\omega)$ | | | | | | | |
| $Y^2(\omega)$ | | | | | | | |
| $XY(\omega)$ | | | | | | | |
| $L[Y \mid X](\omega)$ | | | | | | | |

(a) Fill in the blank entries of the table. In the column to the far right, fill in the expected value of the random variable.

(b) Are the variables correlated or uncorrelated? Are the variables independent or dependent?

(c) Calculate $\mathbb{E}[(Y - L[Y \mid X])^2]$.

## 2 LLSE

We have two bags of balls. The fractions of red balls and blue balls in bag $A$ are $2/3$ and $1/3$ respectively. The fractions of red balls and blue balls in bag $B$ are $1/2$ and $1/2$ respectively. Someone gives you one of the bags (unmarked) uniformly at random. You then draw 6 balls from that same bag with replacement. Let $X_i$ be the indicator random variable that ball $i$ is red. Now, let us define $X = \sum_{1 \le i \le 3} X_i$ and $Y = \sum_{4 \le i \le 6} X_i$. Find $L(Y \mid X)$. *Hint*: Recall that

$$L(Y \mid X) = \mathbb{E}(Y) + \frac{\operatorname{cov}(X,Y)}{\operatorname{var}(X)} \big(X - \mathbb{E}(X)\big).$$

Also remember that covariance is bilinear.

## 3 Number of Ones

In this problem, we will revisit dice-rolling, except with conditional expectation.

(a) If we roll a die until we see a 6, how many ones should we expect to see?

(b) If we roll a die until we see a number greater than 3, how many ones should we expect to see?