

## 1 Working with the Law of Large Numbers

- (a) A fair coin is tossed and you win a prize if there are more than 60% heads. Which is better: 10 tosses or 100 tosses? Explain.
- (b) A fair coin is tossed and you win a prize if there are more than 40% heads. Which is better: 10 tosses or 100 tosses? Explain.
- (c) A coin is tossed and you win a prize if there are between 40% and 60% heads. Which is better: 10 tosses or 100 tosses? Explain.
- (d) A coin is tossed and you win a prize if there are exactly 50% heads. Which is better: 10 tosses or 100 tosses? Explain.

## 2 Markov's Inequality and Chebyshev's Inequality

A random variable  $X$  has variance  $\text{var}(X) = 9$  and expectation  $\mathbb{E}[X] = 2$ . Furthermore, the value of  $X$  is never greater than 10. Given this information, provide either a proof or a counterexample for the following statements.

- (a)  $\mathbb{E}[X^2] = 13$ .
- (b)  $\mathbb{P}[X \leq 1] \leq 8/9$ .
- (c)  $\mathbb{P}[X \geq 6] \leq 9/16$ .

(d)  $\mathbb{P}[X \geq 6] \leq 9/32$ .

### 3 Vegas

On the planet Vegas, everyone carries a coin. Many people are honest and carry a fair coin (heads on one side and tails on the other), but a fraction  $p$  of them cheat and carry a trick coin with heads on both sides. You want to estimate  $p$  with the following experiment: you pick a random sample of  $n$  people and ask each one to flip his or her coin. Assume that each person is independently likely to carry a fair or a trick coin.

1. Given the results of your experiment, how should you estimate  $p$ ?
2. How many people do you need to ask to be 95% sure that your answer is off by at most 0.05?