CS 70 Discrete Mathematics and Probability Theory
Spring 2018 Babak Ayazifar and Satish Rao

DIS 10B

1 Probabilistically Buying Probability Books

Chuck will go shopping for probability books for K hours. Here, K is a random variable and is equally likely to be 1, 2, or 3. The number of books N that he buys is random and depends on how long he shops. We are told that

$$\mathbb{P}[N = n|K = k] = \frac{c}{k}, \qquad \text{for } n = 1, \ldots, k$$

for some constant c.

(a) Compute c.

(b) Find the joint distribution of K and N.

(c) Find the marginal distribution of N.

(d) Find the conditional distribution of K given that $N = 1$.

(e) We are now told that he bought at least 1 but no more than 2 books. Find the conditional mean and variance of K, given this piece of information.

(f) The cost of each book is a random variable with mean 3. What is the expectation of his total expenditure? *Hint:* Condition on events $N = 1, \ldots, N = 3$ and use the total expectation theorem.

2 Continuous Intro

(a) Is

$$f(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

a valid density function? Why or why not? Is it a valid CDF? Why or why not?

(b) Calculate $\mathbb{E}[X]$ and $\text{var}(X)$ for X with the density function

$$f(x) = \begin{cases} 1/\ell, & 0 \le x \le \ell, \\ 0, & \text{otherwise.} \end{cases}$$

(c) Suppose X and Y are independent and have densities

$$f_X(x) = \begin{cases} 2x, & 0 \le x \le 1, \\ 0, & \text{otherwise,} \end{cases}$$

$$f_Y(y) = \begin{cases} 1, & 0 \le y \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

What is their joint distribution? (Hint: for this part and the next, we can use independence in much the same way that we did in discrete probability)

(d) Calculate $\mathbb{E}[XY]$ for the above X and Y.

3 Continuous Computations

Let X be a continuous random variable whose pdf is cx^3 (for some constant c) in the range $0 \le x \le 1$, and is 0 outside this range.

(a) Find c.

(b) Find $\mathbb{P}[1/3 \le X \le 2/3 \mid X \le 1/2]$.

(c) Find $\mathbb{E}(X)$.

(d) Find $\text{var}(X)$.