1 Telebears

Lydia has just started her Telebears appointment. She needs to register for a marine science class and CS 70. There are no waitlists, and she can attempt to enroll once per day in either class or both. The Telebears system is strange and picky, so the probability of enrolling in the marine science class is μ and the probability of enrolling in CS 70 is κ. These events are independent. Let M be the number of days it takes to enroll in the marine science class, and C be the number of days it takes to enroll in CS 70.

(a) What distribution do M and C follow? Are M and C independent?

(b) For some integer $k \geq 1$, what is $\mathbb{P}[C \geq k]$?

(c) For some integer $k \geq 1$, what is the probability that she is enrolled in both classes before day k?

2 Sum of Bernoulli and Geometric Distributions

We know that the sum of i.i.d. Bernoulli random variables follow a binomial distribution. Now, we will consider the sum of i.i.d. geometric random variables.

1. Show that the expectation and variance of the sum of Bernoulli random variables, X_i, matches those of a binomial random variable, B.

2. Find the expectation and variance of the sum of geometric random variables, $G_i \sim \text{Geometric}(p)$.

3. Say you flip a coin until you get k heads. What is the expected number of flips? Variance?

3 Fishy Computations

Use the Poisson distribution to answer these questions:

(a) Suppose that on average, a fisherman catches 20 salmon per week. What is the probability that he will catch exactly 7 salmon this week?

(b) Suppose that on average, you go to Fisherman's Wharf twice a year. What is the probability that you will go at most once in 2018?

(c) Suppose that in March, on average, there are 5.7 boats that sail in Laguna Beach per day. What is the probability there will be *at least* 3 boats sailing throughout the *next two days* in Laguna?

4 Sum of Poisson Variables

Assume that you were given two independent Poisson random variables X_1, X_2. Assume that the first has mean λ_1 and the second has mean λ_2. Prove that $X_1 + X_2$ is a Poisson random variable with mean $\lambda_1 + \lambda_2$.

Hint: Recall the binomial theorem.

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$