

1 How Many Queens?

You shuffle a standard 52-card deck, before drawing the first three cards from the top of the pile. Let X denote the number of queens you draw.

- (a) What is $\mathbb{P}(X = 0)$?
- (b) What is $\mathbb{P}(X = 1)$?
- (c) What is $\mathbb{P}(X = 2)$?
- (d) What is $\mathbb{P}(X = 3)$?
- (e) Do the answers you computed in parts (a) through (d) add up to 1, as expected?
- (f) Compute $\mathbb{E}(X)$ from the definition of expectation.

- (g) Suppose we define indicators X_i , $1 \leq i \leq 3$, where X_i is the indicator variable that equals 1 if the i th card is a queen and 0 otherwise. Compute $\mathbb{E}(X)$.

- (h) Are the X_i indicators independent? Does this affect your solution to part (g)?

2 Quadruply-Repeated Ones

We say that a string of bits has k *quadruply-repeated ones* if there are k positions where four consecutive 1's appear in a row. For example, the string 0100111110 has two quadruply-repeated ones.

What is the expected number of quadruply-repeated ones in a random n -bit string, when $n \geq 3$ and all n -bit strings are equally likely?

3 More Aces in a Deck

There are four aces in a deck. Suppose you shuffle the deck; define the random variables:

X_1 = number of non-ace cards before the first ace

X_2 = number of non-ace cards between the first and second ace

X_3 = number of non-ace cards between the second and third ace

X_4 = number of non-ace cards between the third and fourth ace

X_5 = number of non-ace cards after the fourth ace

1. What is $X_1 + X_2 + X_3 + X_4 + X_5$?
2. Argue that the X_i random variables all have the same distribution. Are they independent?
3. Use the results of the previous parts to compute $\mathbb{E}(X_1)$.

4 Airport Revisited

- (a) Suppose that there are n airports arranged on a circle. There is a plane departing from each airport, and randomly chooses an airport to its left or right and heads towards it. What is the expected number of empty airports after all planes have landed?
- (b) Now suppose that we still have n airports, but instead of sitting on a circle, they form a general graph, where each airport is denoted by a vertex, and an edge between two airports indicates that a flight is permitted between them. There is a plane departing from each airport and randomly chooses a neighboring destination where a flight is permitted. What is the expected number of empty airports after all planes have landed? (Express your answer in terms of $N(i)$ - the set of neighboring airports of airport i , and $\deg(i)$ - the number of neighboring airports of airport i).