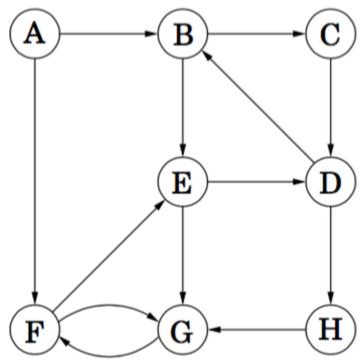
DIS 2A

1 Graph Basics

In the first few parts, you will be answering questions on the following graph G.



(a) What are the vertex and edge sets V and E for graph G?

(b) Which vertex has the highest in-degree? Which vertex has the lowest in-degree? Which vertices have the same in-degree and out-degree?

(c) What are the paths from vertex B to F, assuming no vertex is visited twice? Which one is the shortest path?

(d) Which of the following are cycles in G?

i. $\{(B,C),(C,D),(D,B)\}$
ii. $\{(F,G),(G,F)\}$
iii. $\{(A,B),(B,C),(C,D),(D,B)\}$
iv. $\{(B,C),(C,D),(D,H),(H,G),(G,F),(F,E),(E,D),(D,B)\}$

(e) Which of the following are walks in G?

i. $\{(E,G)\}$
ii. $\{(E,G),(G,F)\}$
iii. $\{(F,G),(G,F)\}$
iv. $\{(A,B),(B,C),(C,D)\}$

v. $\{(E,G),(G,F),(F,G),(G,F)\}$
vi. $\{(E,D),(D,B),(B,E),(E,D),(D,H),(H,G),(G,F)\}$

(f) Which of the following are tours in G?

i. $\{(E,G)\}$
ii. $\{(E,G),(G,F)\}$
iii. $\{(F,G),(G,F)\}$
iv. $\{(E,D),(D,B),(B,E),(E,D),(D,H),(H,G),(G,F)\}$

In the following three parts, let's consider a general undirected graph G with n vertices ($n \geq 3$).

(g) True/False: If each vertex of G has degree at most 1, then G does not have a cycle.

(h) True/False: If each vertex of G has degree at least 2, then G has a cycle.

(i) True/False: If each vertex of G has degree at most 2, then G is not connected.

2 Odd Degree Vertices

Claim: Let $G = (V,E)$ be an undirected graph. The number of vertices of G that have odd degree is even.

Prove the claim above using:

(i) Direct proof (e.g., counting the number of edges in G)

(ii) Induction on $m = |E|$ (number of edges)

(iii) Induction on $n = |V|$ (number of vertices)

(iv) Well-ordering principle

3 Bipartite Graph

A bipartite graph consists of 2 disjoint sets of vertices, such that no 2 vertices in the same set have an edge between them. Consider an undirected bipartite graph with two disjoint sets L,R. Prove that a graph is bipartite if and only if it has no tours of odd length.