CS 70 Discrete Mathematics and Probability Theory
Spring 2018 Satish Rao and Babak Ayazifar

DIS 1B

1 Stable Marriage

Consider the set of men $M = \{1, 2, 3\}$ and the set of women $W = \{A, B, C\}$ with the following preferences.

Men	Women		
1	A	B	C
2	B	A	C
3	A	B	C

Women	Men		
A	2	1	3
B	1	2	3
C	1	2	3

Run the male propose-and-reject algorithm on this example. How many days does it take and what is the resulting pairing? (Show your work.)

2 Stable Marriage

The following questions refer to stable marriage instances with n men and n women, answer True/-False or provide an expression as requested.

(a) For $n = 2$, or any 2-man, 2-woman stable marriage instance, man A has the same optimal and pessimal woman. (True or False.)

(b) In any stable marriage instance, in the pairing the TMA (traditional stable marriage algorithm) produces there is some man who gets his favorite woman (the first woman on his preference list). (True or False.)

(c) In any stable marriage instance with n men and women, if every man has a different favorite woman, a different second favorite, a different third favorite, and so on, and every woman has the same preference list, how many days does it take for TMA to finish? (Form of Answer: An expression that may contain n.)

(d) Consider a stable marriage instance with n men and n women, and where all men have the same preference list, and all women have different favorite men, and different second-favorite men, and so on. How many days does the TMA take to finish? (Form of Answer: An expression that may contain n.)

(e) It is possible for a stable pairing to have a man A and a woman 1 be paired if A is 1's least preferred choice and 1 is A's least preferred choice. (True or False.)

(f) It is possible for a stable pairing to have two couples where each person is paired with their least favorite choice. (True or False.)

(g) If there is a pairing, P, that consists of only pairs from male and female optimal pairings, then it must be stable. In other words, if every pair in P is a pair either in the male-optimal or the female-optimal pairing then P is stable. (True or False.)

3 Good, Better, Best

In a particular instance of the stable marriage problem with n men and n women, it turns out that there are exactly three distinct stable matchings, S_1, S_2, and S_3. Also, each man m has a different partner in the three matchings. Therefore each man has a clear preference ordering of the three matchings (according to the ranking of his partners in his preference list). Now, suppose for man m_1, this order is $S_1 > S_2 > S_3$.

Prove that every man has the same preference ordering $S_1 > S_2 > S_3$.

Hint: Recall that a male-optimal matching always exists and can be generated using TMA. By reversing the roles of TMA, what other matching can we generate?